www.mtg.in | October 2021 | Pages 80 | ₹ 40

SOLVED PAPERS 2021

NEET | JEE SESSION-4 Main

Class XI-XII CBSE warm up! TERM-I

PHYSICS for you

India's #1 PHYSICS MONTHLY FOR JEE (Main & Advanced) & NEET

JEE WORK OUTS

BRAIN MAP CLASS XII

BRUSH UP for NEET/JEE Class XI-XII

MtG

Trust of more than 1 Crore Readers Since 1982

2021100013104

PHYSICS for you

Volume 29 No. 10 October 2021

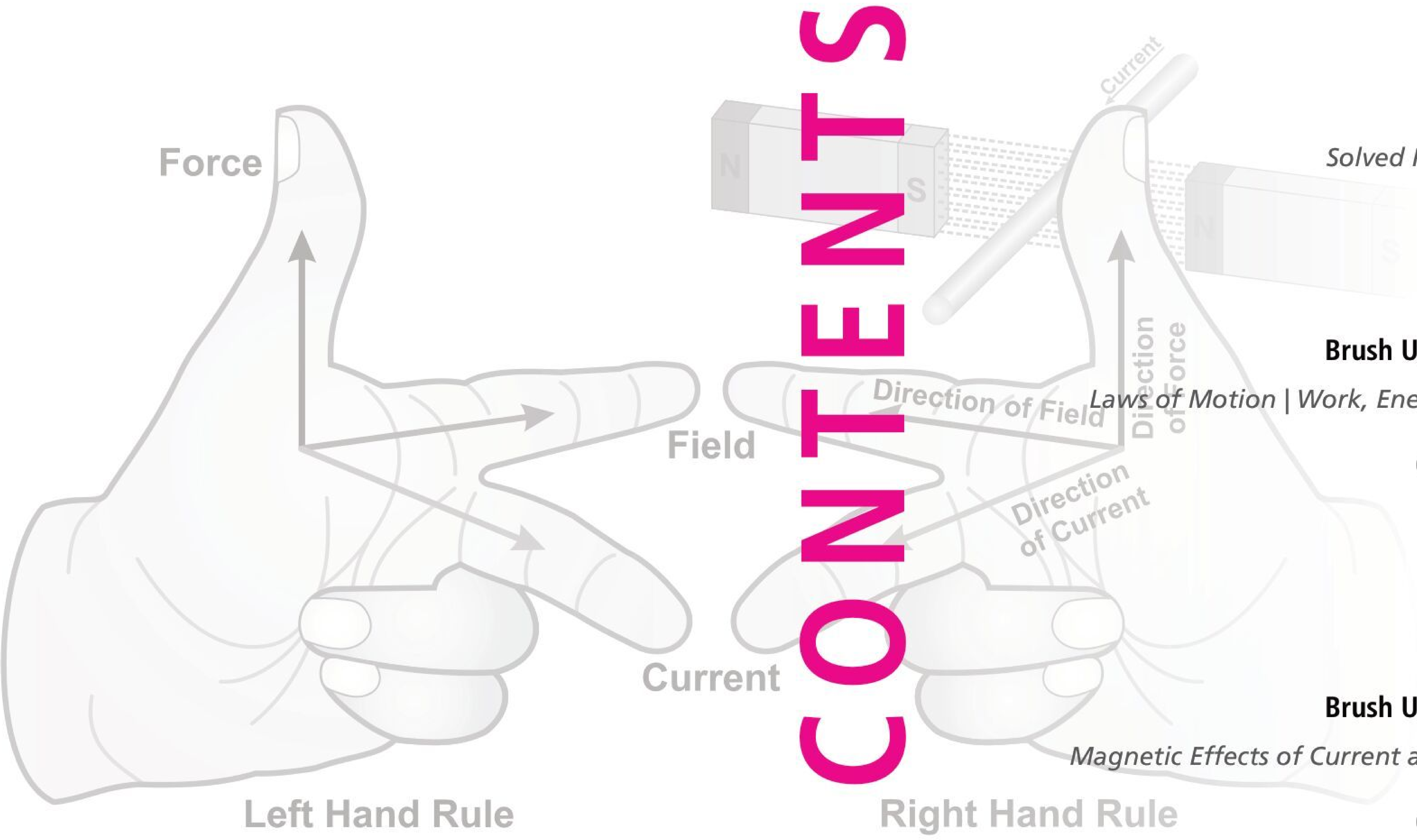
Managing Editor
Mahabir Singh

Editor
Anil Ahlawat

Corporate Office:
Plot 99, Sector 44 Institutional area, Gurugram -122 003 (HR).
Tel : 0124-6601200 e-mail : info@mtg.in website : www.mtg.in

Regd. Office:
406, Taj Apartment, Near Safdarjung Hospital, New Delhi - 110029.

CONTENTS



Printed and Published by Mahabir Singh on behalf of MTG Learning Media Pvt. Ltd. Printed at HT Media Ltd., B-2, Sector-63, Noida, UP-201307 and published at 406, Taj Apartment, Ring Road, Near Safdarjung Hospital, New Delhi - 110029.
Editor : Anil Ahlawat

NEET

SOLVED PAPER 2021

Hurray!! We are happy to inform our readers that out of the 50 questions asked in NEET 2021, more than 90% questions were either exactly same or of similar type from the **MTG Books.** Hurray!!

Here, the references of few are given :

Exam Q. No.	MTG Book	Q. No.	P. No.
1	NEET Champion (XI)	78	305
3	NEET Champion (XII)	60, 61	37
4	NCERT Fingertips (XII)	42	54
6	NEET Champion (XII)	12	120
12	NCERT Fingertips (XII)	63	301
15	NEET Champion (XI)	12	9
17	NEET Guide (XI)	107	144
19	NEET Champion (XII)	43	344
22	NEET Guide (XII)	59	311
25	NEET Champion (XII)	97	128

Exam Q. No.	MTG Book	Q. No.	P. No.
27	NEET Champion (XII)	153	242
28	NEET Champion (XII)	2	212
30	NEET Champion (XII)	219	85
33	NEET Guide (XI)	185	270
35	NEET Guide (XI)	57	141
38	NCERT Fingertips (XI)	58	240
40	NEET Champion (XII)	74	176
41	NCERT Fingertips (XII)	67	471
42	NEET Guide (XI)	56	182
45	NEET Champion (XII)	98	198

and more such questions ......

SECTION-A

1. A body is executing simple harmonic motion with frequency 'n', the frequency of its potential energy is
(a) $4n$ (b) n (c) $2n$ (d) $3n$

2. Polar molecular are the molecules
(a) having a permanent electric dipole moment
(b) having zero dipole moment
(c) acquire a dipole moment only in the presence of electric field due to displacement of charges
(d) acquire a dipole moment only when magnetic field is absent.

3. Column-I gives certain physical terms associated with flow of current through a metallic conductor. Column-II gives some mathematical relations involving electrical quantities. Match column-I and column-II with appropriate relations.

Column-I	Column-II
(A) Drift velocity	(P) $\frac{m}{ne^2\rho}$
(B) Electrical Resistivity	(Q) nev_d
(C) Relaxation Period	(R) $\frac{eE}{m}\tau$
(D) Current Density	(S) $\frac{E}{J}$

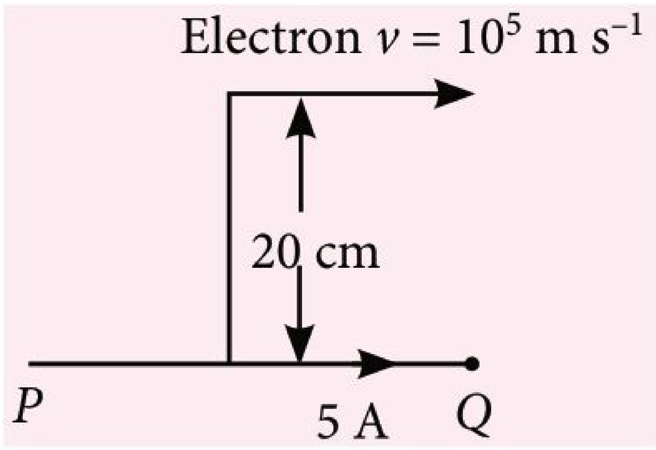
(a) (A)-(R), (B)-(Q), (C)-(S), (D)-(P)
(b) (A)-(R), (B)-(S), (C)-(P), (D)-(Q)
(c) (A)-(R), (B)-(S), (C)-(Q), (D)-(P)
(d) (A)-(R), (B)-(P), (C)-(S), (D)-(Q)

4. Two charged spherical conductors of radius R_1 and R_2 are connected by a wire. Then the ratio of surface charge densities of the spheres (σ_1/σ_2) is
(a) $\frac{R_1^2}{R_2^2}$ (b) $\frac{R_1}{R_2}$ (c) $\frac{R_2}{R_1}$ (d) $\sqrt{\left(\frac{R_1}{R_2}\right)}$

5. A parallel plate capacitor has a uniform electric field '$\vec{E}$' in the space between the plates. If the distance between the plates is 'd' and the area of each plate 'A', the energy stored in the capacitor is (ε_0 = permittivity of free space)

(a) $\dfrac{E^2Ad}{\varepsilon_0}$ (b) $\dfrac{1}{2}\varepsilon_0 E^2$
(c) $\varepsilon_0 EAd$ (d) $\dfrac{1}{2}\varepsilon_0 E^2 Ad$

6. An infinitely long straight conductor carries a current of 5 A as shown. An electron is moving with a speed of 10^5 m s^{-1} parallel to the conductor. The perpendicular distance between the electron and the conductor is 20 cm at an instant. Calculate the magnitude of the force experienced by the electron at that instant.



(a) 8×10^{-20} N (b) 4×10^{-20} N
(c) $8\pi \times 10^{-20}$ N (d) $4\pi \times 10^{-20}$ N

7. If E and G respectively denote energy and gravitational constant, then E/G has the dimensions of
(a) $[M^2][L^{-2}][T^{-1}]$ (b) $[M^2][L^{-1}][T^0]$
(c) $[M][L^{-1}][T^{-1}]$ (d) $[M][L^0][T^0]$

8. A lens of large focal length and large aperture is best suited as an objective of an astronomical telescope since
(a) a large aperture contributes to the quality and visibility of the images
(b) a large area of the objective ensures better light gathering power
(c) a large aperture provides a better resolution
(d) all of the above.

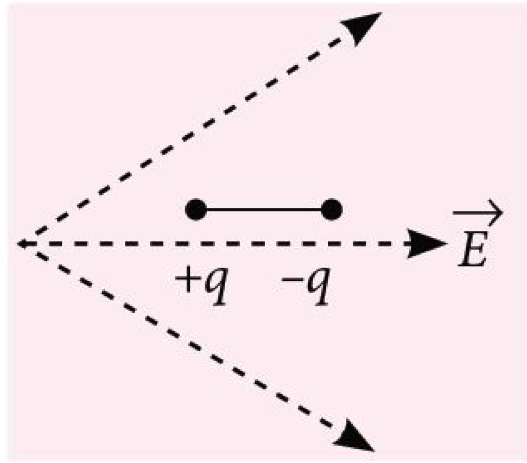
9. Match Column-I and Column-II and choose the correct match from the given choices.

Column-I		**Column-II**
(A) Root mean square speed of gas molecules	(P)	$\frac{1}{3}nm\bar{v}^2$
(B) Pressure exerted by ideal gas	(Q)	$\sqrt{\frac{3RT}{M}}$
(C) Average kinetic energy of a molecule	(R)	5/2 RT
(D) Total internal energy of 1 mole of a diatomic gas	(S)	3/2 K_gT

(a) (A)-(R), (B)-(Q), (C)-(P), (D)-(S)
(b) (A)-(R), (B)-(P), (C)-(S), (D)-(Q)
(c) (A)-(Q), (B)-(R), (C)-(S), (D)-(P)
(d) (A)-(Q), (B)-(P), (C)-(S), (D)-(R)

10. Consider the following statements (A) and (B) and identify the correct answer.
(A) A zener diode is connected in reverse bias, when used as a voltage regulator.
(B) The potential barrier of p-n junction lies between 0.1 V to 0.3 V.
(a) (A) is incorrect but (B) is correct.
(b) (A) and (B) both are correct.
(c) (A) and (B) both are incorrect.
(d) (A) is correct and (B) is incorrect.

11. A dipole is placed in an electric field as shown. In which direction will it move?



(a) Towards the right as its potential energy will increase.
(b) Towards the left as its potential energy will increase.
(c) Towards the right as its potential energy will decrease.
(d) Towards the left as its potential energy will decrease.

12. A convex lens 'A' of focal length 20 cm and a concave lens 'B' of focal length 5 cm are kept along the same axis with a distance 'd' between them. If a parallel beam of light falling on 'A' leaves 'B' as a parallel beam, then the distance 'd' in cm will be
(a) 30 (b) 25 (c) 15 (d) 50

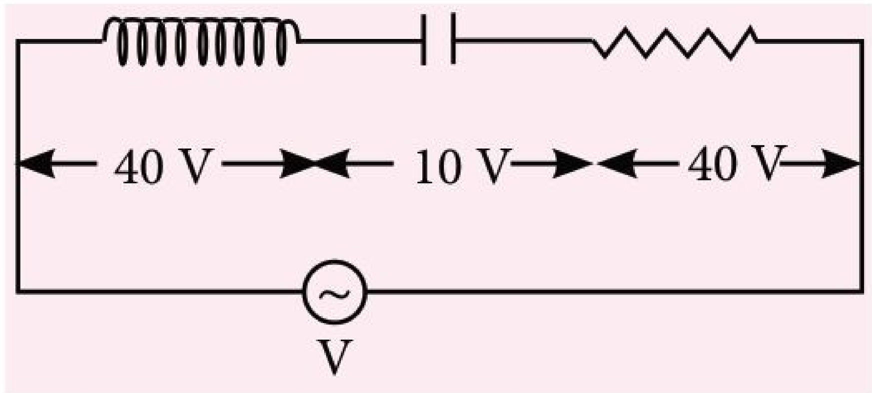
13. The escape velocity from the Earth's surface is v. The escape velocity from the surface of another planet having a radius, four times that of Earth and same mass density is
(a) $4v$ (b) v (c) $2v$ (d) $3v$

14. A radioactive nucleus ${}^A_ZX$ undergoes spontaneous decay in the sequence ${}^A_ZX \rightarrow {}_{Z-1}B \rightarrow {}_{Z-3}C \rightarrow {}_{Z-2}D$, where Z is the atomic number of element X. The possible decay particles in the sequence are
(a) β^-, α, β^+ (b) α, β^-, β^+
(c) α, β^+, β^- (d) β^+, α, β^-

15. A screw gauge gives the following readings when used to measure the diameter of a wire
Main scale reading : 0 mm
Circular scale reading : 52 divisions
Given that 1 mm on main scale corresponds to 100 divisions on the circular scale. The diameter of the wire from the above data is
(a) 0.052 cm (b) 0.52 cm
(c) 0.026 cm (d) 0.26 cm

16. An inductor of inductance L, a capacitor of capacitance C and a resistor of resistance 'R' are connected in series to an ac source of potential difference 'V' volts as shown in figure.
Potential difference across L, C and R is 40 V, 10 V and 40 V, respectively. The amplitude of current

flowing through LCR series circuit is $10\sqrt{2}$ A. The impedance of the circuit is



(a) 5 Ω
(b) $4\sqrt{2}$ Ω
(c) $5/\sqrt{2}$ Ω
(d) 4 Ω

17. A particle is released from height S from the surface of the Earth. At a certain height its kinetic energy is three times its potential energy. The height from the surface of earth and the speed of the particle at that instant are respectively

(a) $\frac{S}{4}, \sqrt{\frac{3gS}{2}}$ (b) $\frac{S}{4}, \frac{3gS}{2}$

(c) $\frac{S}{4}, \frac{\sqrt{3gS}}{2}$ (d) $\frac{S}{2}, \frac{\sqrt{3gS}}{2}$

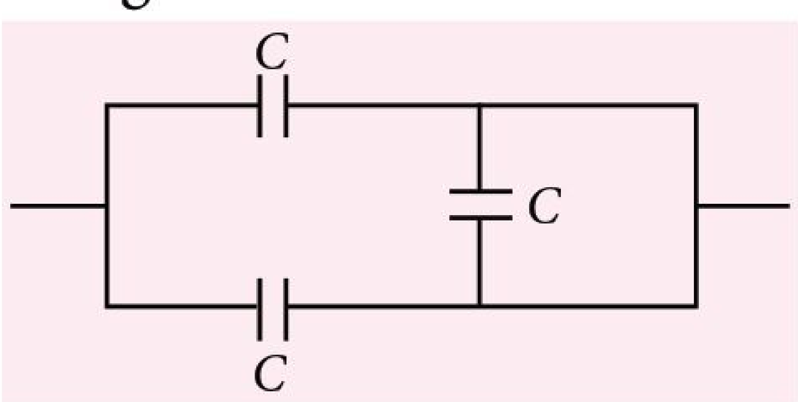
18. A small block slides down on a smooth inclined plane, starting from rest at time $t = 0$. Let S_n be the distance travelled by the block in the interval $t = n - 1$ to $t = n$. Then, the ratio $\frac{S_n}{S_{n+1}}$ is

(a) $\frac{2n}{2n-1}$ (b) $\frac{2n-1}{2n}$ (c) $\frac{2n-1}{2n+1}$ (d) $\frac{2n+1}{2n-1}$

19. The half-life of a radioactive nuclide is 100 hours. The fraction of original activity that will remain after 150 hours would be

(a) $\frac{2}{3\sqrt{2}}$ (b) 1/2 (c) $\frac{1}{2\sqrt{2}}$ (d) $\frac{2}{3}$

20. The equivalent capacitance of the combination shown in the figure is



(a) $3C/2$ (b) $3C$ (c) $2C$ (d) $C/2$

21. The effective resistance of a parallel connection that consists of four wires of equal length, equal area of cross-section and same material is 0.25 Ω. What will be the effective resistance if they are connected in series?

(a) 4 Ω (b) 0.25 Ω (c) 0.5 Ω (d) 1 Ω

22. A nucleus with mass number 240 breaks into two fragments each of mass number 120, the binding energy per nucleon of unfragmented nuclei is 7.6 MeV while that of fragments is 8.5 MeV. The total gain in the binding energy in the process is

(a) 216 MeV (b) 0.9 MeV
(c) 9.4 MeV (d) 804 MeV

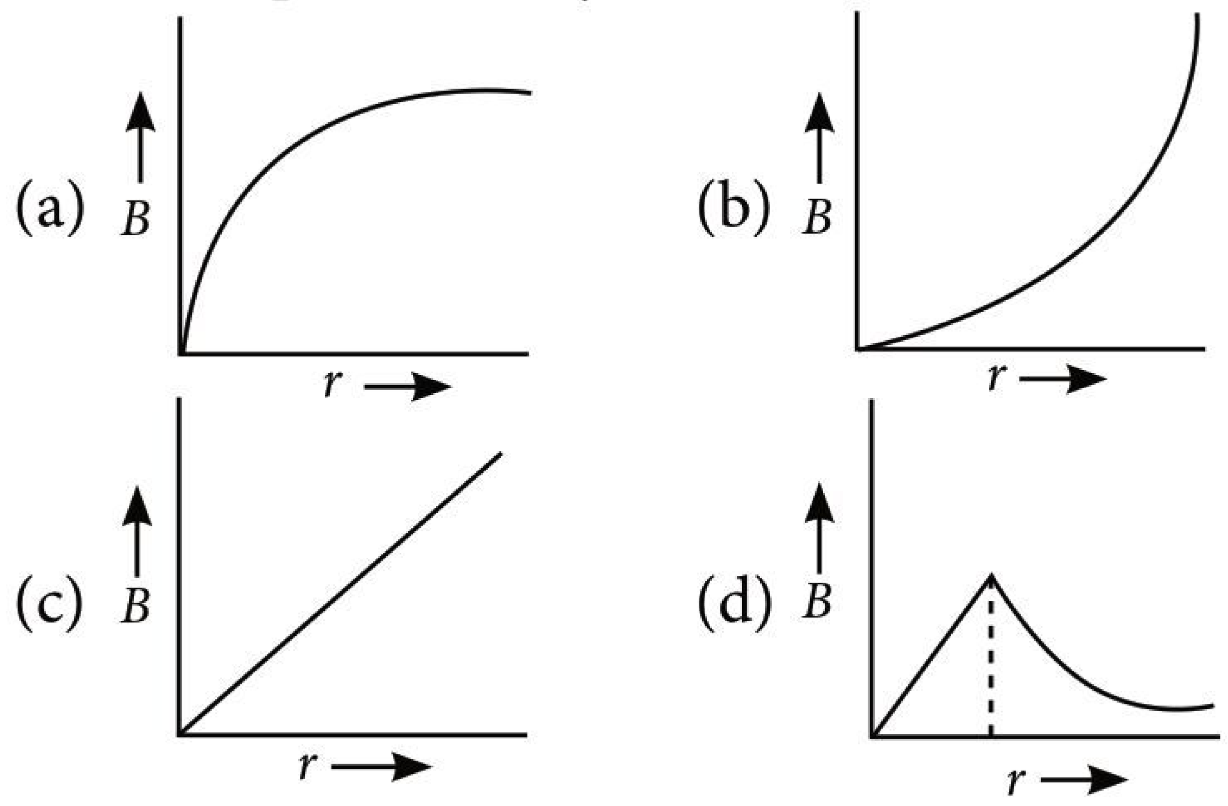
23. The electron concentration in an n-type semiconductor is the same as hole concentration in a p-type semiconductor. An external field (electric) is applied across each of them. Compare the currents in them.

(a) No current will flow in p-type, current will only flow in n-type.
(b) Current in n-type = current in p-type.
(c) Current in p-type > current in n-type.
(d) Current in n-type > current in p-type.

24. The number of photons per second on an average emitted by the source of monochromatic light of wavelength 600 nm, when it delivers the power of 3.3×10^{-3} watt will be ($h = 6.6 \times 10^{-34}$ J s)

(a) 10^{15} (b) 10^{18} (c) 10^{17} (d) 10^{16}

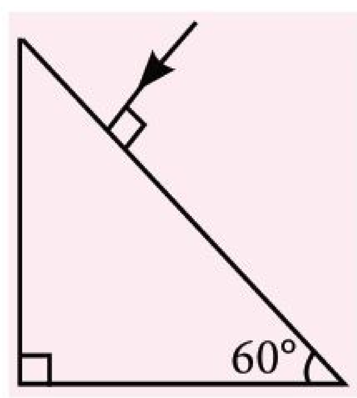
25. A thick current carrying cable of radius 'R' carries current 'I' uniformly distributed across its cross-section. The variation of magnetic field $B(r)$ due to the cable with the distance 'r' from the axis of the cable is represented by



26. The velocity of a small ball of mass M and density d, when dropped in a container filled with glycerine becomes constant after some time. If the density of glycerine is $\frac{d}{2}$, then the viscous force acting on the ball will be

(a) $2Mg$ (b) $\frac{Mg}{2}$ (c) Mg (d) $\frac{3}{2}Mg$

27. Find the value of the angle of emergence from the prism. Refractive index of the glass is $\sqrt{3}$.



(a) 90° (b) 60°
(c) 30° (d) 45°

28. A capacitor of capacitance 'C', is connected across an ac source of voltage V, given by
$V = V_0 \sin\omega t$
The displacement current between the plates of the capacitor, would then be given by

(a) $I_d = V_0\omega C \sin\omega t$ (b) $I_d = V_0\omega C \cos\omega t$

(c) $I_d = \frac{V_0}{\omega C}\cos\omega t$ (d) $I_d = \frac{V_0}{\omega C}\sin\omega t$

29. If force $[F]$, acceleration $[A]$ and time $[T]$ are chosen as the fundamental physical quantities. Find the dimensions of energy.
(a) $[F][A^{-1}][T]$ (b) $[F][A][T]$
(c) $[F][A][T^2]$ (d) $[F][A][T^{-1}]$

30. In a potentiometer circuit a cell of EMF 1.5 V gives balance point at 36 cm length of wire. If another cell of EMF 2.5 V replaces the first cell, then at what length of the wire, the balance point occurs?
(a) 62 cm (b) 60 cm (c) 21.6 cm (d) 64 cm

31. A spring is stretched by 5 cm by a force 10 N. The time period of the oscillations when a mass of 2 kg is suspended by it is
(a) 0.628 s (b) 0.0628 s
(c) 6.28 s (d) 3.14 s

32. For a plane electromagnetic wave propagating in x-direction, which one of the following combination gives the correct possible directions for electric field (E) and magnetic field (B) respectively?
(a) $-\hat{j}+\hat{k}, -\hat{j}+\hat{k}$ (b) $\hat{j}+\hat{k}, \hat{j}+\hat{k}$
(c) $-\hat{j}+\hat{k}, -\hat{j}-\hat{k}$ (d) $\hat{j}+\hat{k}, -\hat{j}-\hat{k}$

33. A cup of coffee cools from 90°C to 80°C in t minutes, when the room temperature is 20°C. The time taken by a similar cup of coffee to cool from 80°C to 60°C at a room temperature same at 20°C is
(a) $\frac{5}{13}t$ (b) $\frac{13}{10}t$ (c) $\frac{13}{5}t$ (d) $\frac{10}{13}t$

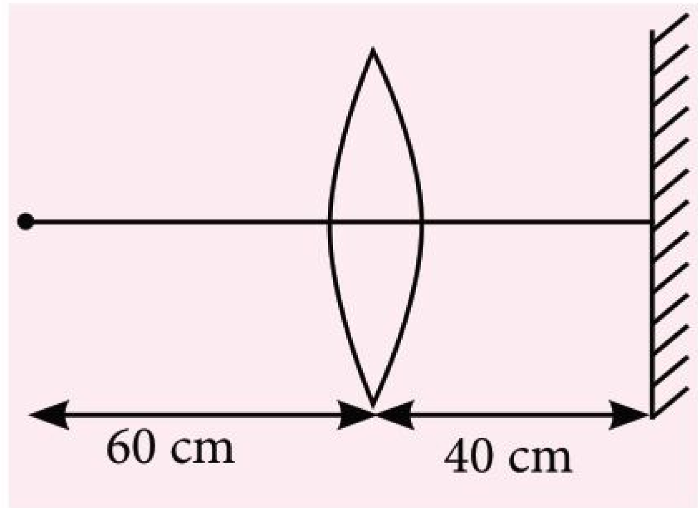
34. An electromagnetic wave of wavelength 'λ' is incident on a photosensitive surface of negligible work function. If 'm' mass is of photoelectron emitted from the surface has de-Broglie wavelength λ_d, then
(a) $\lambda=\left(\frac{2h}{mc}\right)\lambda_d^2$ (b) $\lambda=\left(\frac{2m}{hc}\right)\lambda_d^2$
(c) $\lambda=\left(\frac{2mc}{h}\right)\lambda^2$ (d) $\lambda=\left(\frac{2mc}{h}\right)\lambda_d^2$

35. Water falls from a height of 60 m at the rate of 15 kg s^{-1} to operate a turbine. The losses due to frictional force are 10% of the input energy. How much power is generated by the turbine? ($g = 10$ m s^{-2})
(a) 7.0 kW (b) 10.2 kW
(c) 8.1 kW (d) 12.3 kW

SECTION-B

36. Twenty seven drops of same size are charged at 220 V each. They combine to form a bigger drop. Calculate the potential of the bigger drop.
(a) 1980 V (b) 660 V
(c) 1320 V (d) 1520 V

37. A point object is placed at a distance of 60 cm from a convex lens of focal length 30 cm. If a plane mirror were put perpendicular to the principal axis of the lens and at a distance of 40 cm from it, the final image would be formed at a distance of



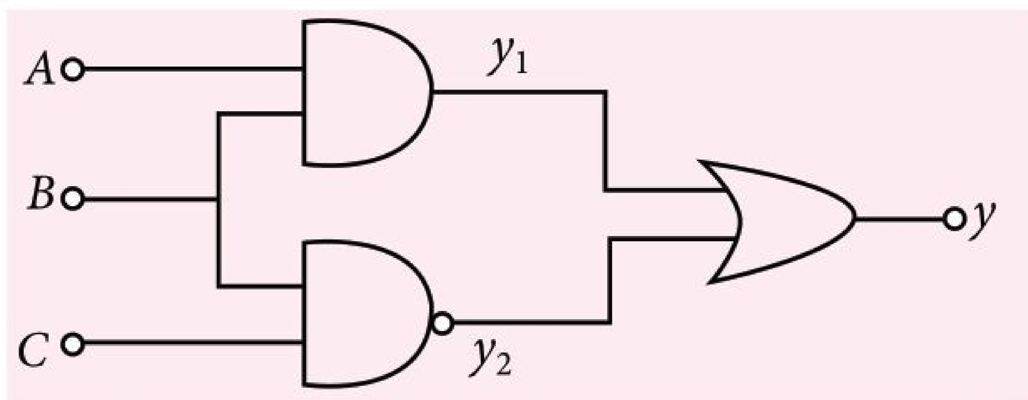
(a) 20 cm from the plane mirror, it would be a virtual image
(b) 20 cm from the lens, it would be a real image
(c) 30 cm from the lens, it would be a real image
(d) 30 cm from the plane mirror, it would be a virtual image.

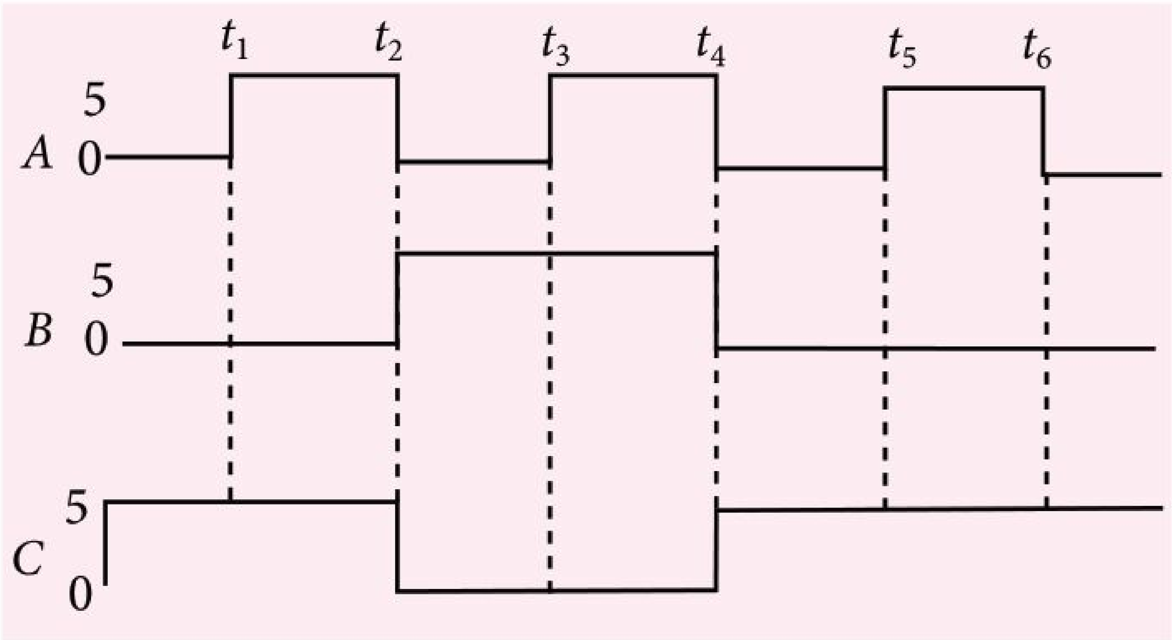
38. A particle of mass 'm' is projected with a velocity $v = kV_e (k < 1)$ from the surface of the earth.
(V_e = escape velocity)
The maximum height above the surface reached by the particle is
(a) $\frac{Rk^2}{1-k^2}$ (b) $R\left(\frac{k}{1-k}\right)^2$
(c) $R\left(\frac{k}{1+k}\right)^2$ (d) $\frac{R^2k}{1+k}$

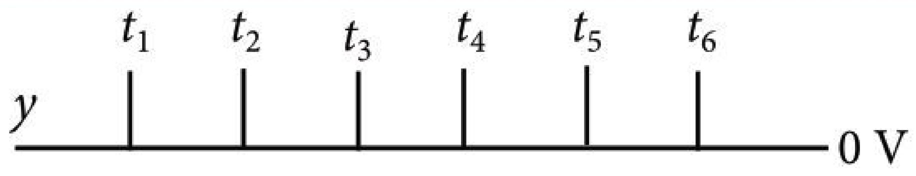
39. In the product
$\vec{F} = q(\vec{v}\times\vec{B}) = q\vec{v}\times(B\hat{i}+B\hat{j}+B_0\hat{k})$
For $q = 1$ and $\vec{v} = 2\hat{i}+4\hat{j}+6\hat{k}$ and
$\vec{F} = 4\hat{i}-20\hat{j}+12\hat{k}$
What will be the complete expression for $\vec{B}$?
(a) $6\hat{i}+6\hat{j}-8\hat{k}$ (b) $-8\hat{i}-8\hat{j}-6\hat{k}$
(c) $-6\hat{i}-6\hat{j}-8\hat{k}$ (d) $8\hat{i}+8\hat{j}-6\hat{k}$

40. Two conducting circular loops of radii R_1 and R_2 are placed in the same plane with their centres coinciding. If $R_1 >> R_2$, the mutual inductance M between them will be directly proportional to
(a) $\frac{R_2^2}{R_1}$ (b) $\frac{R_1}{R_2}$ (c) $\frac{R_2}{R_1}$ (d) $\frac{R_1^2}{R_2}$

41. For the given circuit, the input digital signals are applied at the terminals A, B and C. What would be the output at the terminal y?





(a) 

(b) 

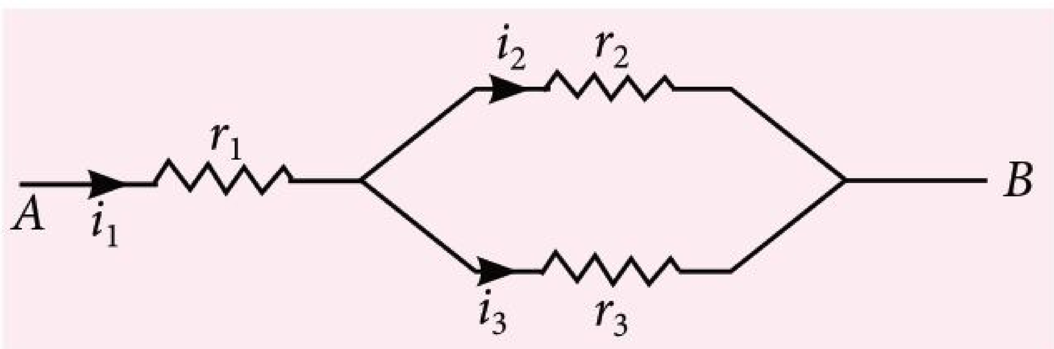
(c) 

(d) 

42. From a circular of mass 'M' and radius 'R' an arc corresponding to a 90° sector is removed. The moment of inertia of the remaining part of the ring about an axis passing through the centre of the ring and perpendicular to the plane of the ring is 'K' times 'MR^2'. Then the value of 'K' is

(a) $\frac{1}{8}$ (b) $\frac{3}{4}$ (c) $\frac{7}{8}$ (d) $\frac{1}{4}$

43. Three resistors having resistances r_1, r_2 and r_3 are connected as shown in the given circuit. The ratio $\frac{i_3}{i_1}$ of currents in terms of resistances used in the circuit is



(a) $\frac{r_2}{r_1 + r_3}$ (b) $\frac{r_1}{r_2 + r_3}$ (c) $\frac{r_2}{r_2 + r_3}$ (d) $\frac{r_1}{r_1 + r_2}$

44. A car starts from rest and accelerates at 5 m s^{-2}. At t = 4 s, a ball is dropped out of a window by a person sitting in the car. What is the velocity and acceleration of the ball at t = 6 s? (Take g = 10 m s^{-2})

(a) $20\sqrt{2}$ m s^{-1}, 10 m s^{-2}

(b) 20 m s^{-1}, 5 m s^{-2}

(c) 20 m s^{-1}, 0 (d) $20\sqrt{2}$ m s^{-1}, 0

45. A step down transformer connected to an ac mains supply of 220 V is made to operate at 11 V, 44 W lamp. Ignoring power losses in the transformer, what is the current in the primary circuit?

(a) 4 A (b) 0.2 A (c) 0.4 A (d) 2 A

46. A ball of mass 0.15 kg is dropped from a height 10 m, strikes the ground and rebounds to the same height. The magnitude of impulse imparted to the ball is (g = 10 m s^{-2}) nearly

(a) 1.4 kg m s^{-1} (b) 0 kg m s^{-1}

(c) 4.2 kg m s^{-1} (d) 2.1 kg m s^{-1}

47. A series LCR circuit containing 5.0 H inductor 8 μF capacitor and 40 Ω resistor is connected to 230 V variable frequency ac source. The angular frequencies of the source at which power transferred to the circuit half the power at the resonant angular frequency are likely to be

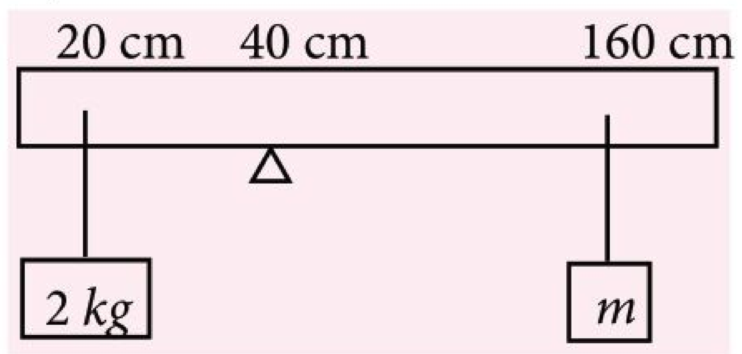
(a) 42 rad s^{-1} and 58 rad s^{-1}

(b) 25 rad s^{-1} and 75 rad s^{-1}

(c) 50 rad s^{-1} and 25 rad s^{-1}

(d) 46 rad s^{-1} and 54 rad s^{-1}

48. A uniform rod of length 200 cm and mass 500 g is balanced on a wedge placed at 40 cm mark. A mass of 2 kg is suspended from the rod at 20 cm and another unknown mass 'm' is suspended from the rod at 160 cm mark as shown in the figure. Find the value of 'm' such that the rod is in equilibrium. (g = 10 m s^{-2})



(a) $\frac{1}{12}$ kg (b) $\frac{1}{2}$ kg (c) $\frac{1}{3}$ kg (d) $\frac{1}{6}$ kg

49. A particle moving in a circle of radius R with a uniform speed taken a time T to complete one revolution.

if this particle were projected with the same speed at an angle 'θ' to the horizontal, the maximum height attained by it equal 4R. The angle of projection θ, is then given by

(a) $\theta = \sin^{-1}\left(\frac{2gT^2}{\pi^2 R}\right)^{1/2}$ (b) $\theta = \cos^{-1}\left(\frac{gT^2}{\pi^2 R}\right)^{1/2}$

(c) $\theta = \cos^{-1}\left(\frac{\pi^2 R}{gT^2}\right)^{1/2}$ (d) $\theta = \sin^{-1}\left(\frac{\pi^2 R}{gT^2}\right)^{1/2}$

50. A uniform conducting wire of length 12a and resistance 'R' is wound up as a current carrying coil in the shape of

(i) an equilateral triangle of side 'a'.

(ii) a square of side 'a'.

The magnetic dipole moment of the coil in each case respectively are

(a) $4Ia^2$ and $3Ia^2$ (b) $\sqrt{3}Ia^2$ and $3Ia^2$

(c) $3Ia^2$ and Ia^2 (d) $3Ia^2$ and $4Ia^2$

SOLUTIONS

1. (c) : The equation for the SHM is

$y = A\sin\omega t = A\sin(2\pi nt)$ ...(i)

where, A is amplitude, ω is angular frequency and y is displacement at time t.

The formula of potential energy is

$$U = \frac{1}{2}kA^2\sin^2\omega t$$

$$U = \frac{1}{2}kA^2\left(\frac{1-\cos 2\omega t}{2}\right) = \frac{1}{2}kA^2\left[\frac{1-\cos 2\pi(2n)t}{2}\right] \quad ...(ii)$$

$$\left(\because \sin^2\theta = \frac{1-\cos 2\theta}{2}\right)$$

From eqn. (i) and (ii), we get

Frequency of potential energy = $2n$

2. (a) : The polar molecules are the molecules have one end slightly positive and other end is slightly negatively charged separated by some distance.

So, they have permanent electric dipole moment.

3. (b) : The formula of drift velocity is, $v_d = \frac{eE}{m}\tau$

Current density, $J = \frac{I}{A} = \frac{neAv_d}{A} = nev_d$

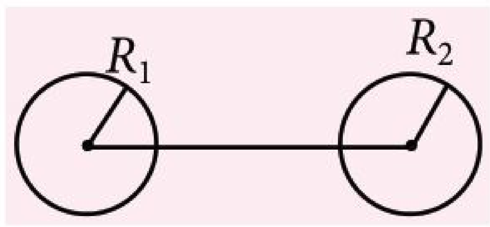
Resistivity is, $\rho = \frac{m}{ne^2\tau} \Rightarrow \tau = \frac{m}{ne^2\rho}$

Resistance is, $R = \frac{V}{I}$; $\rho\frac{l}{A} = \frac{El}{I} \Rightarrow \rho = \frac{EA}{I} = \frac{E}{J}$

where, E = electric field, A = area of cross section

e = electronic charge, n = number of density of electrons, τ = relaxation time.

4. (c) : When the two spheres are connected by conducting wire, the potential of both the spheres becomes same.



$$V_1 = V_2 \Rightarrow \frac{kq_1}{R_1} = \frac{kq_2}{R_2} ; \frac{q_1}{q_2} = \frac{R_1}{R_2} \quad ...(i)$$

Ratio of surface charge densities

$$\frac{\sigma_1}{\sigma_2} = \frac{\frac{Q}{4\pi R_1^2}}{\frac{Q}{4\pi R_2^2}} = \frac{R_2^2}{R_1^2}\times\frac{q_1}{q_2} = \left(\frac{R_2}{R_1}\right)^2\times\frac{R_1}{R_2} \quad \text{(From (i))}$$

$$\Rightarrow \frac{\sigma_1}{\sigma_2} = \frac{R_2}{R_1}$$

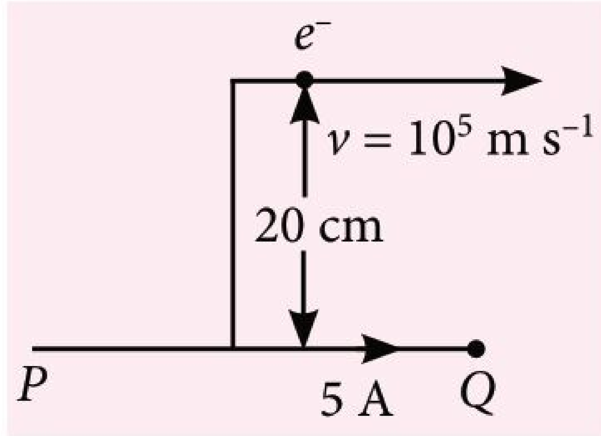
5. (d) : The capacitance of a parallel plate capacitor of plate area A and separation d is given by $C = \frac{\varepsilon_0 A}{d}$

The potential is given by $V = Ed$

where E is electric field.

Energy, $U = \frac{1}{2}CV^2 = \frac{1}{2}\times\frac{\varepsilon_0 A}{d}\times E^2d^2$; $U = \frac{1}{2}\varepsilon_0 E^2 Ad$

6. (a) : The magnetic field due to a straight wire at the location of electron is



$$B = \frac{\mu_0}{4\pi}\cdot\frac{2I}{r}$$

$$B = 10^{-7}\times\frac{2\times 5}{0.2} = 50\times 10^{-7}\ \text{T}\ (\hat{k})$$

The direction is given by right hand thumb rule.

The force on charged particle moving in magnetic field is, $\vec{F} = q(\vec{v}\times\vec{B})$

$\vec{F} = -1.6\times 10^{-19}\times(10^5\hat{i}\times 50\times 10^{-7}\hat{k})$; $\vec{F} = 8\times 10^{-20}\ \text{N}\ \hat{j}$

7. (b) : Dimensions of energy $[E] = [ML^2T^{-2}]$

Dimensions of gravitational constant $[G] = [M^{-1}L^3T^{-2}]$

So, the dimensions of $\left[\frac{E}{G}\right] = \frac{[ML^2T^{-2}]}{[M^{-1}L^3T^{-2}]} = [M^2L^{-1}T^0]$

$= [M^2][L^{-1}][T^0]$

8. (d) : As the focal length is large so, it enhances the magnifying power of telescope. The large aperture or diameter of lens helps in collecting large amount of light from the object so that the image formed is bright.

The resolving power of telescope is, $\text{R.P.} = \frac{D}{1.22\lambda}$

where, D is diameter or aperture of lens and λ is wavelength of light used.

So, all options are correct.

9. (d) : The rms velocity is, $v_{rms} = \sqrt{\frac{3RT}{M}}$

where, R is a gas constant, M = molecular mass, T = absolute temperature. So, A→ Q

Pressure exerted by ideal gas is $\frac{1}{3}mn\bar{v}^2$, where m is mass of each molecule, n = number of molecules, $\bar{v}^2$ = rms speed. So, B → P.

Average kinetic energy of a molecule $\frac{3}{2}k_BT$

where, k_B = Boltzmann's constant, T = absolute temperature. So, C → S.

Total internal energy of 1 mole of a diatomic gas, $U = \frac{5}{2}RT$. So, D → R.

10. (d) : Yes, zener diode is connected in reverse bias as it is used as voltage regulator.

The potential barrier of p-n junction

For Ge; V_o = 0.3 V ; Si; V_o = 0.7 V

11. (c) : Electric field is always direct from high potential to low potential. For the given situation the electric potential is decreasing from left to right therefore, potential energy of the dipole will also decrease. Thus dipole will move from towards the right.

12. (c) : Given : $f_1 = 20$ cm, $f_2 = -5$ cm
Equivalent focal length, $f = \infty$
(As the rays are parallel)
By using Newton's displacement formula

$$\frac{1}{f} = \frac{1}{f_1} + \frac{1}{f_2} - \frac{d}{f_1 f_2}$$

$$\frac{1}{\infty} = \frac{1}{20} - \frac{1}{5} - \frac{d}{20(-5)}$$

$$0 = \frac{5 - 20 + d}{20 \times 5} \Rightarrow d = 15 \text{ cm}$$

13. (a) : The formula of escape velocity is $v = \sqrt{\frac{2GM}{R}}$
where, G = gravitational constant, M = mass and R = radius.

Mass, $M = \frac{4}{3}\pi R^3 \rho$ where, ρ is density of planet

$$v = \sqrt{\frac{2 \times G}{R} \times \frac{4}{3}\pi R^3 \rho}$$

$$v = \sqrt{\frac{8G\pi}{3} R^2 \rho} \quad \text{...(i)}$$

Let the escape velocity on planet is v'.
For planet, $R' = 4R$, $\rho' = \rho$

$$v' = \sqrt{\frac{8G\pi}{3} R'^2 \rho'} \quad \text{...(ii)}$$

Divide eqn. (i) by (ii), we get

So, $\frac{v'}{v} = \sqrt{\frac{R'^2}{R^2} \times \frac{\rho'}{\rho}} \Rightarrow \frac{v'}{v} = \sqrt{\frac{16R^2}{R^2} \times \frac{\rho}{\rho}} = 4$

$v' = 4v$

14. (d) : A radioactive nucleus ${}^{A}_{Z}X$ undergoes spontaneous decay as shown here,

$$_{Z}X^{A} \xrightarrow{\text{I}} {}_{Z-1}\text{B} \xrightarrow{\text{II}} {}_{Z-3}\text{C} \xrightarrow{\text{III}} {}_{Z-2}\text{D}$$

I step : atomic number decreased by 1 unit, so it is β^+ decay.
II step : atomic number decreased by 2 units, so it is α decay (α).
III step : atomic number increased by 1 unit, so it is β^- decay.

15. (a) : Given : Pitch = 1 mm, Number of division on circular scale, $N = 100$

Least count, L.C. $= \frac{\text{Pitch}}{N} = \frac{1}{100} = 0.01$ mm

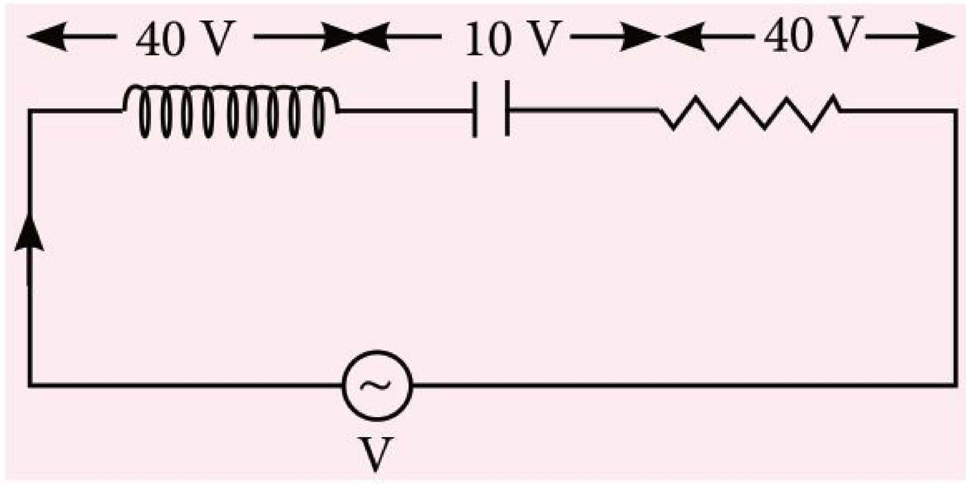
So, diameter, d = main scale reading + (circular scale reading × least count)

$d = 0 + 52 \times 0.01$; $d = 0.52$ mm ; $d = 0.052$ cm

16. (a) : Given, $V_L = 40$ V, $V_C = 10$ V, $V_R = 40$ V

$$V_{\text{rms}} = \sqrt{V_R^2 + (V_L - V_C)^2}$$

$$V_{\text{rms}} = \sqrt{40^2 + (40 - 10)^2} = 50 \text{ V}$$



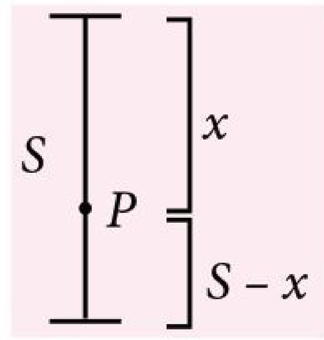
$I_0 = 10\sqrt{2}$ A

$I_{\text{rms}} = \frac{I_0}{\sqrt{2}} = 10$ A

Impedance,

$$Z = \frac{V_{\text{rms}}}{I_{\text{rms}}} = \frac{50}{10} = 5\ \Omega$$

17. (a) : Let the point is P where K.E. is three times of P.E.
Let the height fall is x and speed at P is v.



$\text{KE}_p = 3\ \text{PE}_p = 3mg\ (S - x)$ ...(i)
Use conservation of energy
PE at top = PE at P + KE at P
$mgS = mg(S - x) + \text{KE}_p$
$\text{KE}_p = mgS - mg(S - x)$ ...(ii)
From (i) and (ii)
$3mg(S - x) = mgS - mg(S - x)$
$4mg\ (S - x) = mgS$

$4S - 4x = S \Rightarrow x = \frac{3S}{4}$

So, $S - x = S - \frac{3S}{4} = \frac{S}{4}$

Now, $\text{KE}_p = \frac{1}{2}mv^2 = 3mg(S - x)$

$$\frac{v^2}{2} = 3g\left(S - \frac{3S}{4}\right) \Rightarrow \frac{v^2}{2} = 3g \times \frac{S}{4} \Rightarrow v = \sqrt{\frac{3gS}{2}}$$

18. (c) : The acceleration is $a = g\sin\theta$
Initial velocity, $u = 0$

Distance travelled in n^{th} second, $S_{n^{\text{th}}} = u + \frac{1}{2}a(2n - 1)$

$$S_{n^{\text{th}}} = \frac{1}{2}a(2n - 1) \quad \text{...(i)}$$

$$S_{(n+1)^{\text{th}}} = \frac{1}{2}a[2(n + 1) - 1] \quad \text{...(ii)}$$

On dividing eqn (i) by (ii), we get

$$\therefore \frac{S_{n^{th}}}{S_{(n+1)^{th}}} = \frac{(2n - 1)}{(2n + 1)}$$

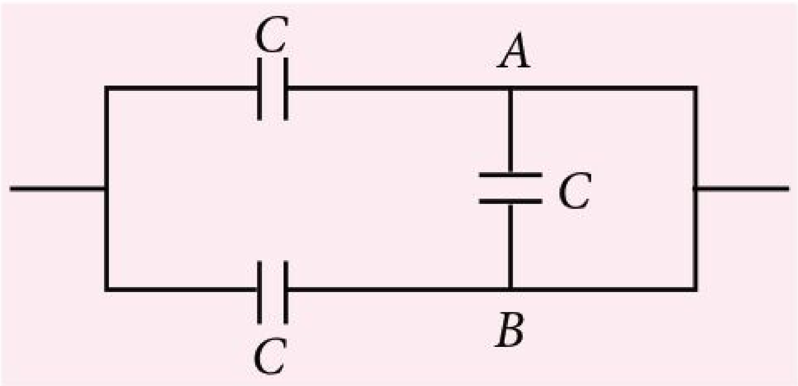
19. (c) : Half-life, $T_{1/2} = 150$ hr
Time, $T = 100$ hr

By the formula of radioactivity, $\frac{N}{N_0} = \left(\frac{1}{2}\right)^n$

where, n = number of half lives $= \frac{150}{100} = \frac{3}{2}$

So, $\frac{N}{N_0} = \left(\frac{1}{2}\right)^{3/2} = \frac{1}{2\sqrt{2}}$

where, N_0 is initial activity and N is remaining activity.

20. (c) : 

Here, AB arm is short, so the two capacitors C and C in parallel, $C_{eq} = C + C = 2C$

21. (a) : Given that, four wires of same material, equal area of cross-section and equal length.
Resistance of each wire is 'R'.
For parallel combination, $R_{eq} = 0.25\ \Omega$
When wires are connected in parallel,

$$\frac{1}{R_{eq}} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3} + \frac{1}{R_4}\ ;\ \frac{1}{R_{eq}} = \frac{1}{R} + \frac{1}{R} + \frac{1}{R} + \frac{1}{R}$$

$$\frac{1}{0.25} = \frac{4}{R} \Rightarrow R = 1\,\Omega$$

So, resistance of each wire is 1 Ω.
Now, wires are connected in series. Then, equivalent resistance is
$R_{eq} = R_1 + R_2 + R_3 + R_4$; $R_{eq} = 1\,\Omega + 1\,\Omega + 1\,\Omega + 1\,\Omega = 4\,\Omega$
Hence, the equivalent resistance in series will be 4 ohm.

22. (a) : Let the parent nucleus be X which breaks into two fragments Y.
$^{240}X \rightarrow 2\,^{120}Y$
Binding energy X, B.E.$_X$ = 240 × 7.6 = 1824 MeV
Binding energy of Y, B.E.$_Y$ = 120 × 8.5 = 1020 MeV
Energy released, $E = 2(BE)_Y - (BE)_X$
= 2 × (1020) – 1824 = 216 MeV

23. (d) : As electrons are majority charge carriers in n-type semiconductor. So when we apply electric field, current will be more in n-type semiconductor compare to p-type semiconductor.

24. (d) : Given, Power = 3.3×10^{-3} Watt
Wavelength, $\lambda = 600$ nm $= 600 \times 10^{-9}$ m
Number of photons emitted per second is

$$n = \frac{P\lambda}{hc} = \frac{3.3\times10^{-3}\times600\times10^{-9}}{6.6\times10^{-34}\times3\times10^{8}}$$

$n = 10^{16}$

25. (d) : By Ampere's circuital law,

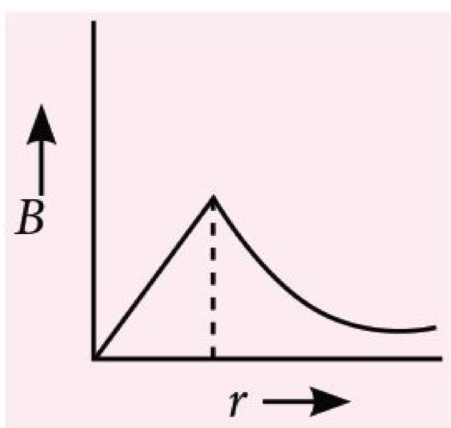
$$\oint \vec{B}\cdot d\vec{l} = \mu_0 \times \text{current enlosed by path}$$

$$\Rightarrow\ B\cdot 2\pi r = \mu_0 \times \frac{Ir^2}{a^2}\ \Rightarrow\ B = \frac{\mu_0 Ir}{2\pi d^2}\quad (\text{for } r < a)$$

At surface of $r = a$, so $B = \dfrac{\mu_0 I}{2\pi a}$

$\therefore\ B = \dfrac{\mu_0 I}{2\pi r}$ (for $r > a$)

The variation of magnetic field with distance 'r' from the axis is given by

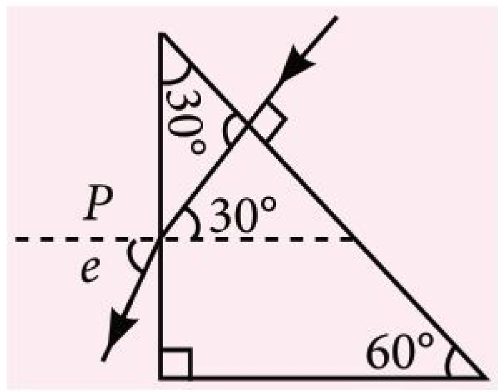


26. (b) : Given, Mass of ball = M
Density of ball = d
Density of glycerine $= \dfrac{d}{2}$
Viscous force = weight – buoyant force $= vdg - v\left(\dfrac{d}{2}\right)g$

$$= vdg\left(1 - \frac{1}{2}\right) = \frac{vdg}{2} \quad \text{...(i)}$$

Also mass, $M = vd$
So, from equation (i),
Viscous force $= \dfrac{Mg}{2}$

27. (b) : Let $\angle e$ is angle of emergence and $\angle i$ is angle of incidence.
Form Snell's law at point P



$$\frac{\sin i}{\sin r} = \frac{\mu_a}{\mu_g}$$

$\angle i = 30°$

So, $\dfrac{\sin 30°}{\sin e} = \dfrac{\mu_a}{\mu_g}$

$\sin e = \dfrac{\sqrt{3}}{1} \times \dfrac{1}{2}$ or $e = 60°$

28. (b) : The instantaneous voltage, $V = V_0 \sin\omega t$
Displacement current is given by

$$I_d = \frac{CdV}{dt}$$

$$I_d = \frac{Cd}{dt}(V_0 \sin\omega t)$$

$\Rightarrow\ I_d = CV_0\omega \cos\omega t$

29. (c) : Let for energy, $[E] = F^{\alpha}A^{\beta}T^{\gamma}$
or $[M^1L^2T^{-2}] = [MLT^{-2}]^{\alpha}[LT^{-2}]^{\beta}[T]^{\gamma}$
$M^1L^2T^{-2} = M^{\alpha}L^{\alpha+\beta}T^{-2\alpha-2\beta+\gamma}$
Comparing from both sides, $\alpha = 1$
$\alpha + \beta = 2 \Rightarrow \beta = 1$
$-2\alpha - 2\beta + \gamma = -2 \Rightarrow \gamma = 2$
$\therefore$ Energy = $[F][A][T^2]$

30. (b) : Given, Balancing point $l_1 = 36$ cm when voltage applied is 1.5 V.
$V_2 = 2.5$ V, l_2 = ?
Potential gradient, $k = \dfrac{V}{l} = \dfrac{1.5}{36}$ V/cm
If l_2 be the balancing length in second case, then

$$\frac{1.5}{36} \times l_2 = 2.5 \quad \text{or} \quad l_2 = 60 \text{ cm}$$

31. (a) : Force constant of spring, $k = \dfrac{F}{x}$
Here, $F = 10$ N, $x = 5$ cm

$$k = \frac{10}{0.05} = 200 \text{ N/m}$$

If the mass of 2 kg is suspended by the spring, then period of oscillation is

$$T = 2\pi\sqrt{\frac{m}{k}}$$

$$T = 2\pi\sqrt{\frac{2}{200}} = 2\pi \times 0.1 \quad ; T = 0.628 \text{ s}$$

32. (c) : As $\vec{E}$ and $\vec{B}$ are perpendicular to each other, their dot product must be zero.

$$\vec{E}\cdot\vec{B} = 0$$

Also, the wave is propagating along x-axis *i.e.* cross product of $\vec{E}\times\vec{B}$ is along $\hat{i}$.

Out of given options, only $(-\hat{j}+\hat{k}), (-\hat{j}-\hat{k})$ follow the above conditions.

33. (c) : Initial temperature (T_i) = 90°C

Final temperature (T_f) = 80°C

Room temperature (T_0) = 20°C

Let time taken be t minutes.

According to Newton's law of cooling,

Rate of cooling $\frac{dT}{dt} = K\left[\frac{(T_i + T_f)}{2} - T_0\right]$

$$\frac{(T_f - T_i)}{t} = K\left[\frac{(90+80)}{2} - 20\right]$$

$$\frac{90-80}{t} = K[65] \Rightarrow K = \frac{10}{65t}$$

In 2nd condition,

Initial temperature, T_i = 80°C

Final temperature, T_f = 60°C

Let time taken be t' minutes.

Then, $\frac{(80-60)}{t'} = \frac{10}{65t}\left[\frac{(80+60)}{2} - 20\right]$

$$\frac{20}{t'} = \frac{10}{65t}(50) \Rightarrow t' = \frac{13}{5}t$$

34. (d) : As work function is negligible, therefore kinetic energy of emitted electron = Energy of incident photon

i.e., $\frac{1}{2}mv^2 = h\upsilon \Rightarrow \frac{p^2}{2m} = \frac{hc}{\lambda} \Rightarrow p = \sqrt{\frac{2mhc}{\lambda}}$

de-broglie wavelength of emitted electrons is

$$\lambda_d = \frac{h}{p} = \frac{h}{\sqrt{\frac{2mhc}{\lambda}}}$$

$$\Rightarrow \lambda_d = \sqrt{\frac{h\lambda}{2mc}} \Rightarrow \lambda = \left(\frac{2mc}{h}\right)\lambda_d^2$$

35. (c) : Given, h = 60 m

Now, water falls at the rate of 15 kg s^{-1}

i.e., $\frac{m}{t}$ = 15 kg s^{-1}

g = 10 m s^{-2}

As loss due to friction is 10%, therefore only 90% of input energy is used to generate power.

$$\therefore \quad P = gh\frac{m}{t}\times\frac{90}{100}$$

$$P = 10\times 60\times 15\times\frac{90}{100} = 8100 \text{ W} \Rightarrow \text{P} = 8.1 \text{ kW}$$

36. (a) : Here : n = 27

Potential, V = 220 V

Potential at the surface of a solid charged sphere

$$V = \frac{1}{4\pi\varepsilon_0}\frac{q}{r}$$

$$\frac{4}{3}\pi r^3 \times n = \frac{4}{3}\pi r'^3 \text{ or } r' = rn^{1/3} \quad \text{...(i)}$$

Potential of bigger drop,

$$V_n = \frac{1}{4\pi\varepsilon_0}\cdot\frac{nq}{r'} \Rightarrow V_n = \frac{1}{4\pi\varepsilon_0}\frac{nq}{rn^{1/3}} \quad \text{(using (i))}$$

$$\therefore \quad V_n = Vn^{2/3} \quad \text{...(ii)}$$

Putting the values of n and V in equation (ii), we get

$V_n = 220 \times (27)^{2/3}$ or V_n = 1980 V

37. (a) : Distance of object, u = – 60 cm

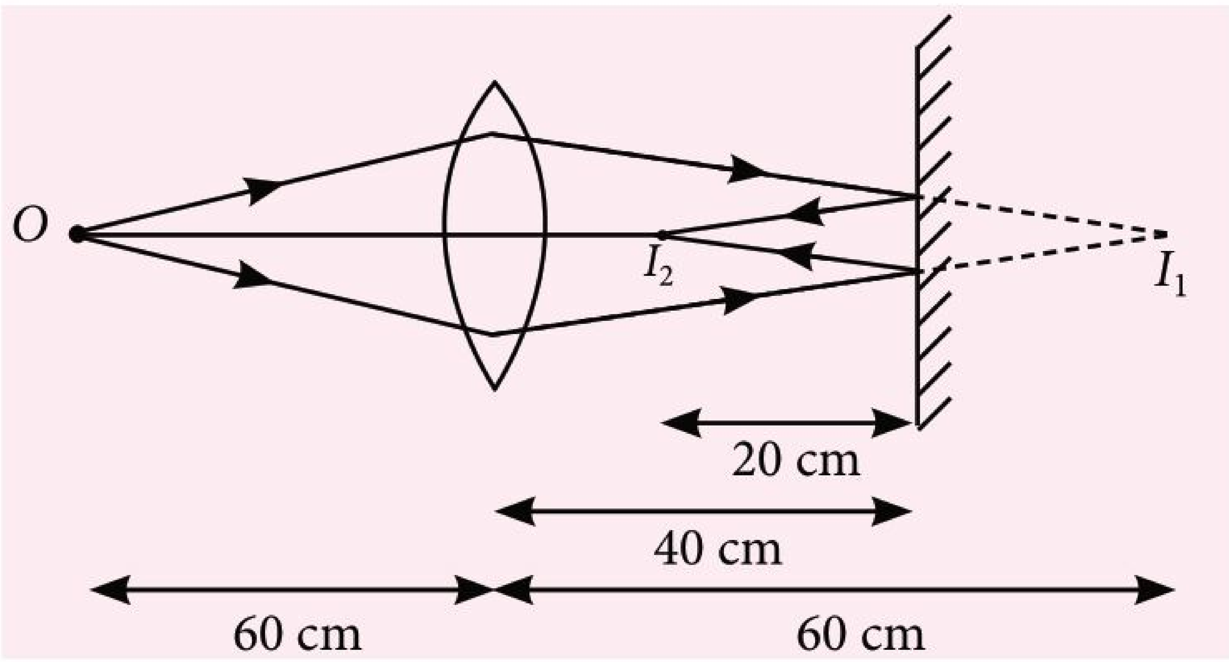
Focal length, f = 30 cm

Let the image formed by lens is at a distance v.

Using lens equation

$$\frac{1}{v} - \frac{1}{u} = \frac{1}{f} \Rightarrow \frac{1}{v} + \frac{1}{60} = \frac{1}{30} \Rightarrow \frac{1}{v} = \frac{1}{60} \Rightarrow v = 60 \text{ cm}$$

So, image formed behind the mirror at a distance (60 – 40) = 20 cm.



For second refraction from convex lens

u = –20 cm, v = ?, f = 30 cm

$$\frac{1}{v} - \frac{1}{u} = \frac{1}{f} \Rightarrow \frac{1}{v} + \frac{1}{20} = \frac{1}{30} \Rightarrow \frac{1}{v} = \frac{1}{30} - \frac{1}{20}$$

$\Rightarrow v$ = – 60 cm

Thus final image is virtual and at a distance 60 – 40 = 20 cm from plane mirror.

38. (a) : The particle is fired vertically upwards from the Earth's surface with a velocity v and reaches a height h.

Energy of the particle at the surface of the Earth is

$$E_i = \frac{1}{2}mv^2 - \frac{GM_E m}{R_E}$$

Energy of the particle at a height h

$$E_f = -\frac{GM_E m}{R_E + h}$$

($\because$ At height h, velocity of the particle is zero.)

According to law of conservation of energy,

$$E_i = E_f$$

$$\frac{1}{2}mv^2 - \frac{GM_E m}{R_E} = -\frac{GM_E m}{R_E + h}$$

$$\frac{1}{2}mv^2 = GM_E m\left[\frac{1}{R_E} - \frac{1}{R_E + h}\right] = \frac{GM_E mh}{(R_E)(R_E + h)}$$

$$\frac{1}{2}mv^2 = \frac{mghR_E}{R_E + h} \quad ...(i) \qquad \left(\because g = \frac{GM_E}{R_E^2}\right)$$

As per question,

$$v = kv_e = k\sqrt{2gR_E} \quad ...(ii) \qquad (\because v_e = V_e = \sqrt{2gR_E})$$

Using (i) and (ii), we get

$$h = \frac{R_E k^2}{1 - k^2}$$

If $R_E = R$, then $h = \frac{Rk^2}{1 - k^2}$

39. (c) : Given : $\vec{v} = 2\hat{i} + 4\hat{j} + 6\hat{k}$ and $q = 1$;

$$\vec{B} = B\hat{i} + B\hat{j} + B_0\hat{k}$$

$$\vec{F} = 4\hat{i} - 20\hat{j} + 12\hat{k}$$

Now, $\vec{v} \times \vec{B} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 2 & 4 & 6 \\ B & B & B_0 \end{vmatrix}$

$$= \hat{i}(4B_0 - 6B) - \hat{j}(2B_0 - 6B) + \hat{k}(2B - 4B)$$

Force $\vec{F} = q(\vec{v} \times \vec{B})$

$$4\hat{i} - 20\hat{j} + 12\hat{k} = 1\ [(4B_0 - 6B)\hat{i} - (2B_0 - 6B)\hat{j} - 2B\hat{k}]$$

By comparison

$4B_0 - 6B = 4$ ...(i)

$-2B = 12$

$\Rightarrow B = -6$ ...(ii)

From eqn. (i) and (ii), $4B_0 - 6(-6) = 4$

$\Rightarrow 4B_0 + 36 = 4 \Rightarrow 4B_0 = -32 \Rightarrow B_0 = -8$

So, $\vec{B} = -6\hat{i} - 6\hat{j} - 8\hat{k}$

40. (a) : Let a current I_1 flows through the outer circular coil of radius R_1.

The magnetic field at the centre of the outer loop is

$$B_1 = \frac{\mu_0 I_1}{2R_1}$$

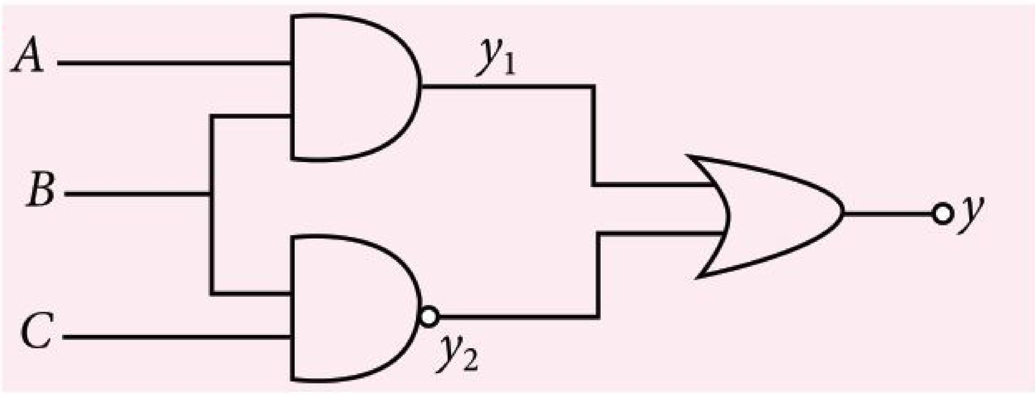
As the inner loop of radius R_2 placed coaxially therefore, B_1 may be taken constant over its cross sectional area.

Hence flux associated with inner loop is

$$\phi_2 = B_1\pi R_2^2 = \frac{\mu_0 I_1}{2R_1}\pi R_2^2$$

As $M = \frac{\phi_2}{I_1} = \frac{\mu_0 \pi R_2^2}{2R_1} \quad \therefore \ M \propto \frac{R_2^2}{R_1}$

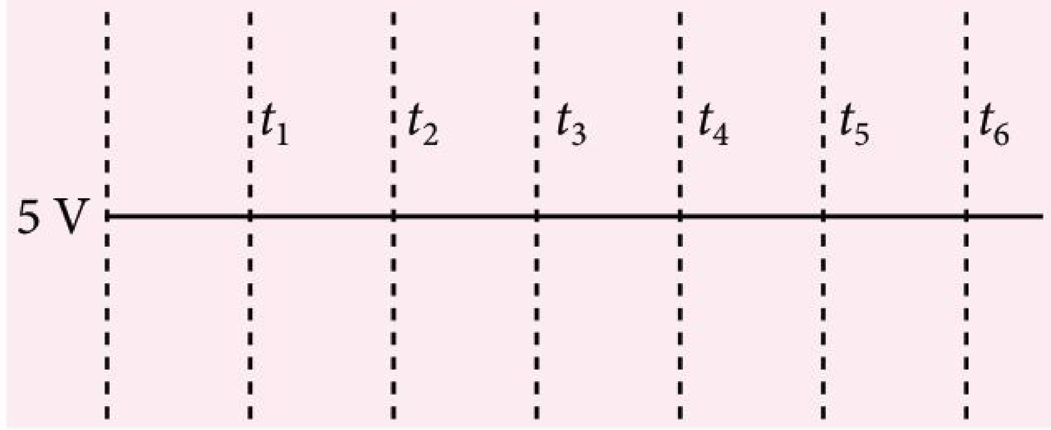
41. (c) :



Output of combination of logic gates is given as

$$y = AB + \overline{BC}$$

A	B	C	y_1	y_2	$y = AB + \overline{BC}$
0	0	0	0	1	1
1	0	0	0	1	1
0	1	0	0	1	1
0	0	1	0	1	1
1	1	0	0	1	1
1	0	1	0	1	1
1	1	1	1	0	1



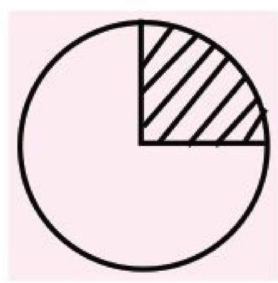
The output (y) can be represented as above.

42. (b) : Here : M is the mass and R is the radius of the ring.

Moment of inertia of a ring, $I = MR^2$

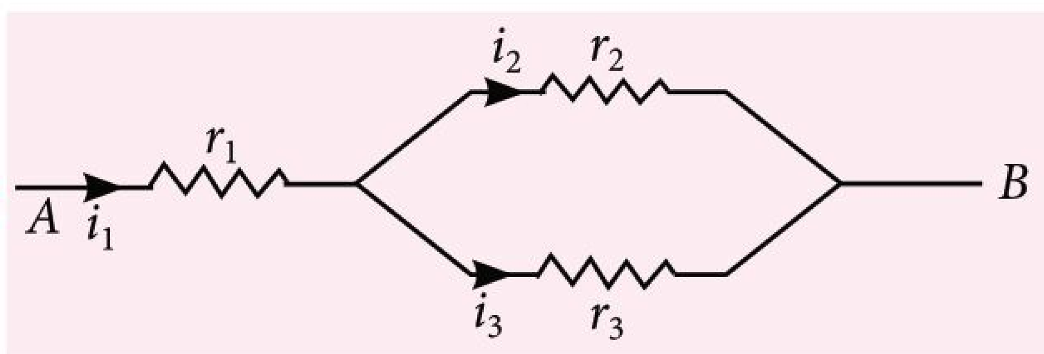
If 90° sector is removed, the moment of inertia of the remaining part of the ring is

$$I = MR^2 - \frac{MR^2}{4} \Rightarrow KMR^2 = \frac{3}{4}MR^2$$



Here $K = \frac{3}{4}$

43. (c) :



By KCL

$i_1 = i_2 + i_3$ ...(i)

Voltage is same in r_2 and r_3 as they are in parallel

$$i_2 r_2 = i_3 r_3$$

$$i_2 = \frac{i_3 r_3}{r_2} \quad ...(ii)$$

From eqn. (i) and (ii), $i_1 = \frac{i_3 r_3}{r_2} + i_3$

or $i_1 = \frac{i_3 r_3 + i_3 r_2}{r_2} = i_3\left(\frac{r_2 + r_3}{r_2}\right)$; $\frac{i_3}{i_1} = \frac{r_2}{r_2 + r_3}$

44. (a) : Given : $u_x = 0$, $a_x = 5$ m s^{-2}
At t = 4s ball is dropped
$a_y = g = 10$ m s^{-2}
Let v_x is the velocity of ball at the time t = 4 s.
$v_x = u_x + a_x t = 0 + 5 \times 4 = 20$ m s^{-1}
v_y is the velocity of ball after leaving at t = 6 s
$v_y = u_y + a_y t' = 0 + g \times 2 = 20$ m s^{-1}
Resultant velocity of ball at t = 6 s

$$v = \sqrt{v_x^2 + v_y^2} = \sqrt{20^2 + 20^2}$$
$$v = 20\sqrt{2} \text{ m s}^{-1}$$

Acceleration of the ball will be due to gravity *i.e.*, $a = 10$ m s^{-2}.

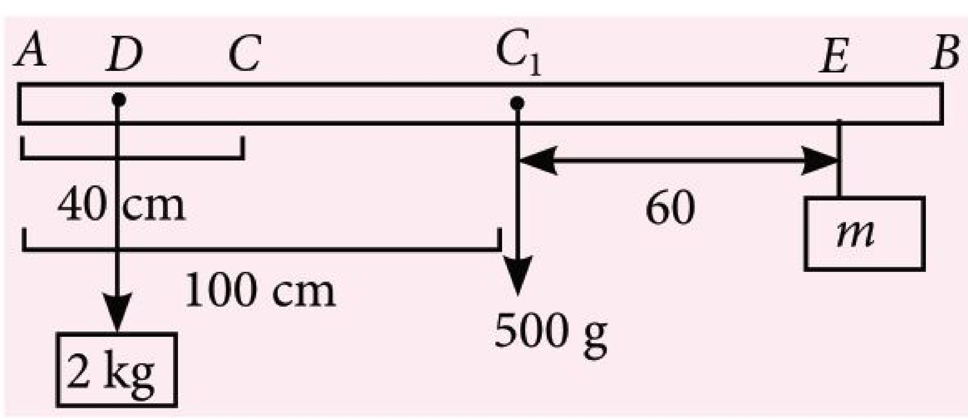
45. (b) : Here P = 44 W, V_p = 220 V, V_s = 11 V
Power, $P = V_s I_s$
$44 = I_s \Rightarrow I_s = 4$ A
Now, $V_p I_p = V_s I_s$
$220 \times I_p = 11 \times 4$
$$I_P = \frac{11 \times 4}{220} = \frac{4}{20} = 0.2 \text{ A}$$

46. (c) : Mass, m = 0.15 kg
h_1 = 10 m, h_2 = 10 m, g = 10 m s^{-2}
The velocity at the time of strike while going downwards
$$v = -\sqrt{2gh_1}$$
and when move up
$$v' = \sqrt{2gh_2}$$
Impulse = change in momentum
$mv' - mv = 2mv$ (As $h_1 = h_2$)
$$= 2 \times 0.15 \times \sqrt{2 \times 10 \times 10} = 4.2 \text{ kg m s}^{-1}$$

47. (d) : $\omega_r = \frac{1}{\sqrt{LC}} = \frac{1}{\sqrt{5 \times 80 \times 10^{-6}}} = 50 \text{ rad s}^{-1}$

Power transferred = half of power at resonance
So, frequencies at which power transferred is half
$= \omega_r \pm \Delta\omega$
$$\Delta\omega = \frac{R}{2L} = \frac{40}{2 \times 5} = 4 \text{ rad/s}$$
So, range is $\omega_r \pm \Delta\omega = 50 \pm 4 = 54$ rad s^{-1} and 46 rad s^{-1}

48. (a) : Here, AD = 20 cm
AE = 160 cm
g = 10 m s^{-2}
Take moments about C



Clockwise moment = Anticlockwise moment
$2 \times g \times DC = 0.50 \times g \times CC_1 + m \times g \times CE$
or $2 \times (40 - 20) = 0.500\,(100 - 40) + m \times (60 + 60)$
or $40 - 30 = 120\,m$
or $m = \frac{1}{12}$ kg

49. (a) : We know, $T = \frac{2\pi R}{u}$
$$\Rightarrow u = \frac{2\pi R}{T}$$
Here : $H_{max} = 4R$
$$H_{max} = \frac{u^2 \sin^2\theta}{2g} \quad \text{...(i)}$$
Putting the value of u and H_{max} in equation (i), we get
$$4R = \left(\frac{2\pi R}{T}\right)^2 \times \frac{\sin^2\theta}{2g} \Rightarrow \theta = \sin^{-1}\left(\frac{2gT^2}{\pi^2 R}\right)^{1/2}$$

50. (b) : For an equilateral triangle of side a :
As total length of the wire = $12a$
So, number of turns, $n = \frac{12a}{3a} = 4$
Magnetic moment of coil, $m = nIA$ ...(i)
Area of triangle, $A = \frac{\sqrt{3}}{4}a^2$
From equation (i),
$$m = 4I\frac{\sqrt{3}}{4}a^2 \quad \therefore \quad m = \sqrt{3}Ia^2$$
For a square of side a :
Area, $A = a^2$
Number of turns, $n = \frac{12a}{4a} = 3$
Magnetic moment, $m = nIA$
$= 3I(a^2) = 3Ia^2$

SOLVED PAPER

Held on 26^{th} August, Morning Shift

JEE MAIN 2021

SECTION-A (MULTIPLE CHOICE QUESTIONS)

1. The rms speeds of the molecules of Hydrogen, Oxygen and Carbondioxide at the same temperature are v_H, v_O and v_C respecetively then
   (a) $v_C > v_O > v_H$ (b) $v_H = v_O > v_C$
   (c) $v_H > v_O > v_C$ (d) $v_H = v_O = v_C$

2. The initial mass of a rocket is 1000 kg. Calculate at what rate the fuel should be burnt so that the rocket is given an acceleration of 20 m s^{-2}. The gases come out at a relative speed of 500 m s^{-1} with respect to the rocket. [Use g = 10 m s^{-2}]
   (a) 10 kg s^{-1} (b) 60 kg s^{-1}
   (c) 500 kg s^{-1} (d) 6.0×10^2 kg s^{-1}

3. A particular hydrogen like ion emits radiation of frequency 2.92×10^{15} Hz when it makes transition from $n = 3$ to $n = 1$. The frequency in Hz of radiation emitted in transition from $n = 2$ to $n = 1$ will be
   (a) 6.57×10^{15} (b) 2.46×10^{15}
   (c) 0.44×10^{15} (d) 4.38×10^{15}

4. Inside a uniform spherical shell
   (A) the gravitational field is zero.
   (B) the gravitational potential is zero.
   (C) the gravitational field is same everywhere.
   (D) the gravitational potentail is same everywhere.
   (E) all of the above
   (a) (A), (C) and (D) only
   (b) (A), (B) and (C) only
   (c) (B), (C) and (D) only
   (d) (E) only

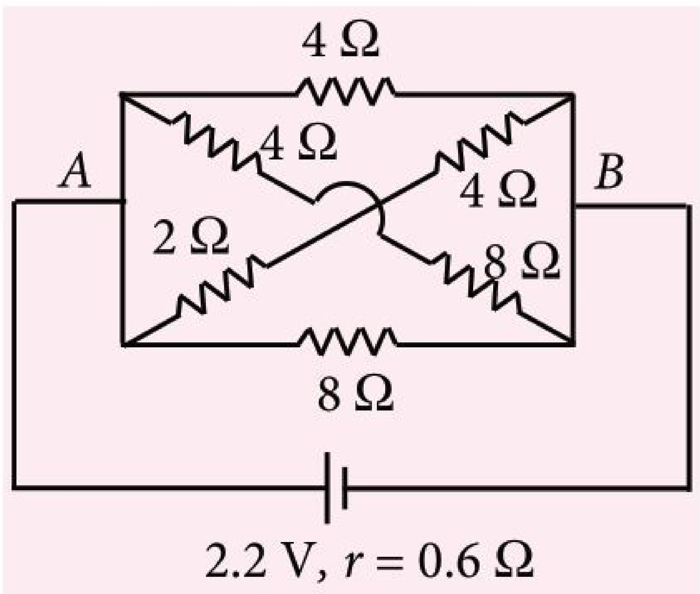
5. Car B overtakes another car A at a relative speed of 40 m s^{-1}. How fast will the image of car B appear to move in the mirror of focal length 10 cm fitted in car A, when the car B is 1.9 m away from the car A?
   (a) 0.1 m s^{-1} (b) 0.2 m s^{-1}
   (c) 40 m s^{-1} (d) 4 m s^{-1}

6. If E, L, M and G denote the quantities as energy, angular momentum, mass and constant of gravitation respectively, then the dimensions of P in the formula $P = EL^2M^{-5}G^{-2}$ are
   (a) $[M^1 L^1 T^{-2}]$ (b) $[M^0 L^1 T^0]$
   (c) $[M^{-1} L^{-1} T^2]$ (d) $[M^0 L^0 T^0]$

7. An electric appliance supplies 6000 J per min heat to the system. If the system delivers a power of 90 W, how long it would take to increase the internal energy by 2.5×10^3 J ?
   (a) 2.5×10^1 s (b) 2.5×10^2 s
   (c) 2.4×10^3 s (d) 4.1×10^1 s

8. A series LCR circuit driven by 300 V at a frequency of 50 Hz contains a resistance R = 3 kΩ, an inductor of inductive reactance $X_L = 250\pi\,\Omega$ and an unknown capacitor. The value of capacitance to maximize the average power should be (Take $\pi^2 = 10$)
   (a) 400 μF (b) 4 μF
   (c) 40 μF (d) 25 μF

9. In the given figure, the emf of the cell is 2.2 V and if internal resistance is 0.6 Ω. Calculate the power dissipated in the whole circuit.

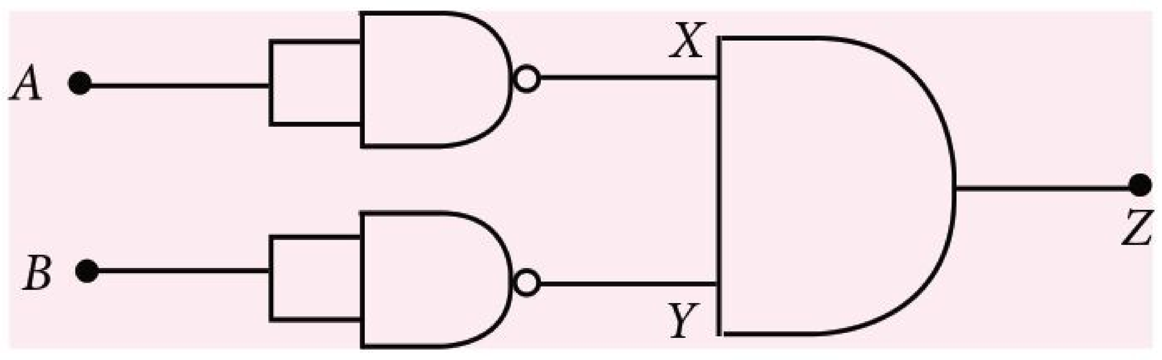


   (a) 1.32 W (b) 4.4 W
   (c) 0.65 W (d) 2.2 W

10. In a photoelectric experiment, ultraviolet light of wavelength 280 nm is used with lithium cathode having work function ϕ = 2.5 eV. If the wavelength

of incident light is switched to 400 nm, find out the change in the stopping potential.
($h = 6.63 \times 10^{-34}$ J s, $c = 3 \times 10^8$ m s^{-1})
(a) 1.1 V (b) 0.6 V
(c) 1.3 V (d) 1.9 V

11. Idenify the logic operation carried out by the given circuit.



(a) OR (b) AND
(c) NAND (d) NOR

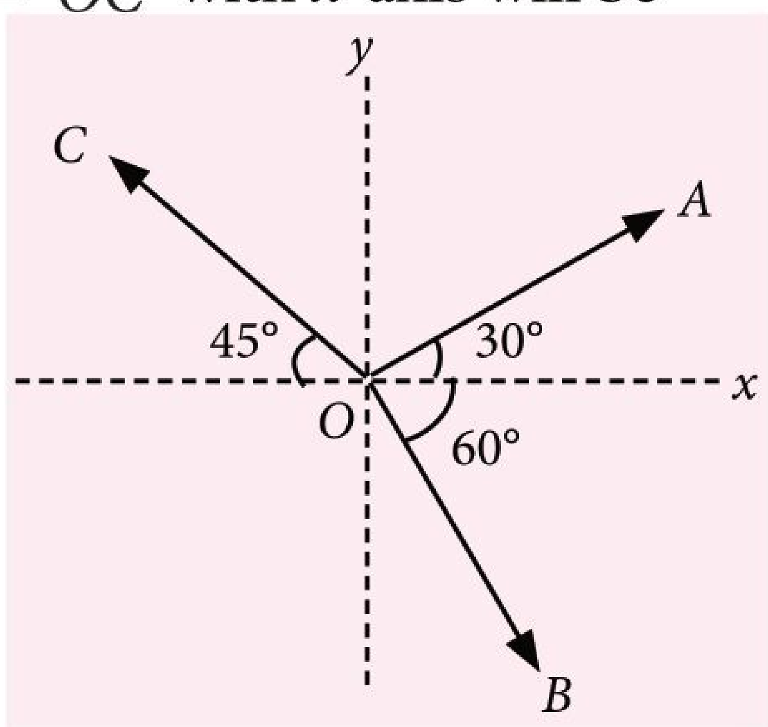
12. What equal length of an iron wire and a copper -nickel alloy wire, each of 2 mm diameter connected parallel to give an equivalent resistance of 3 Ω? (Given : resistivities of iron and copper-nickel alloy wire are 12 μΩ cm and 51 μΩ cm respectively)
(a) 97 m (b) 110 m
(c) 90 m (d) 82 m

13. An inductor coil stores 64 J of magnetic field energy and dissipates energy at the rate of 640 W when a current of 8 A is passed through it. If this coil is joined across an ideal battery, find the time constant of the circuit in seconds.
(a) 0.4 (b) 0.2
(c) 0.125 (d) 0.8

14. In a screw gauge, fifth division of the circular scale coincides with the reference line when the ratchet is closed. There are 50 divisions on the circular scale, and the main scale moves by 0.5 mm on a complete rotation. For a particular observation the reading on the main scale is 5 mm and the 20th division of the circular scale coincides with reference line. Calculate the true reading.
(a) 5.20 mm (b) 5.00 mm
(c) 5.15 mm (d) 5.25 mm

15. Two narrow bores of diameter 5.0 mm and 8.0 mm are joined together to form a U-shaped tube open at both ends. If this U-tube contains water, what is the difference in the level of two limbs of the tube. [Take surface tension of water $T = 7.3 \times 10^{-2}$ N m^{-1}, angle of contact = 0, g = 10 m s^{-2} and density of water = 1.0×10^3 kg m^{-3}].
(a) 5.34 mm (b) 3.62 mm
(c) 2.19 mm (d) 4.97 mm

16. The magnitude of vectors $\overrightarrow{OA}, \overrightarrow{OB}$ and $\overrightarrow{OC}$ in the given figure are equal. The direction of $\overrightarrow{OA} + \overrightarrow{OB} - \overrightarrow{OC}$ with x-axis will be



(a) $\tan^{-1}\dfrac{(\sqrt{3}-1+\sqrt{2})}{(1+\sqrt{3}-\sqrt{2})}$ (b) $\tan^{-1}\dfrac{(1-\sqrt{3}-\sqrt{2})}{(1+\sqrt{3}+\sqrt{2})}$

(c) $\tan^{-1}\dfrac{(\sqrt{3}-1+\sqrt{2})}{(1-\sqrt{3}+\sqrt{2})}$ (d) $\tan^{-1}\dfrac{(1+\sqrt{3}-\sqrt{2})}{(1-\sqrt{3}-\sqrt{2})}$

17. The material filled between the plates of a parallel plate capacitor has resistivity 200 Ω m. The value of capacitance of the capacitor is 2 pF. If a potential difference of 40 V is applied across the plates of the capacitor, then the value of leakage current flowing out of the capacitor is (Given the value of relative permitivity of material is 50)
(a) 0.9 mA (b) 9.0 mA
(c) 9.0 μA (d) 0.9 μA

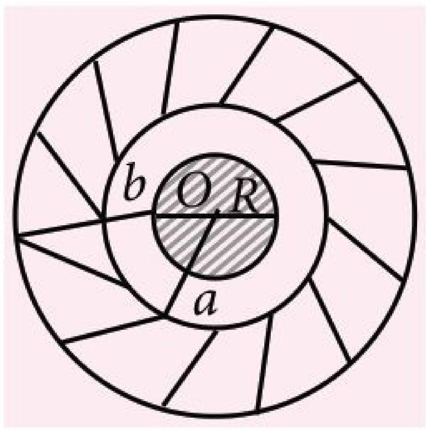
18. **Statement-I :** By doping silicon semiconductor with pentavalent material, the electrons density increases.
Statement-II : The n-type semiconductor has net negative charge.
In the light of the above statements, choose the most appropriate answer for the statements from the options given below.
(a) Both statement I and statement II are false.
(b) Statement I is true but statement II is false.
(c) Statement I is false but statement II is true.
(d) Both statement I and statement II are true.

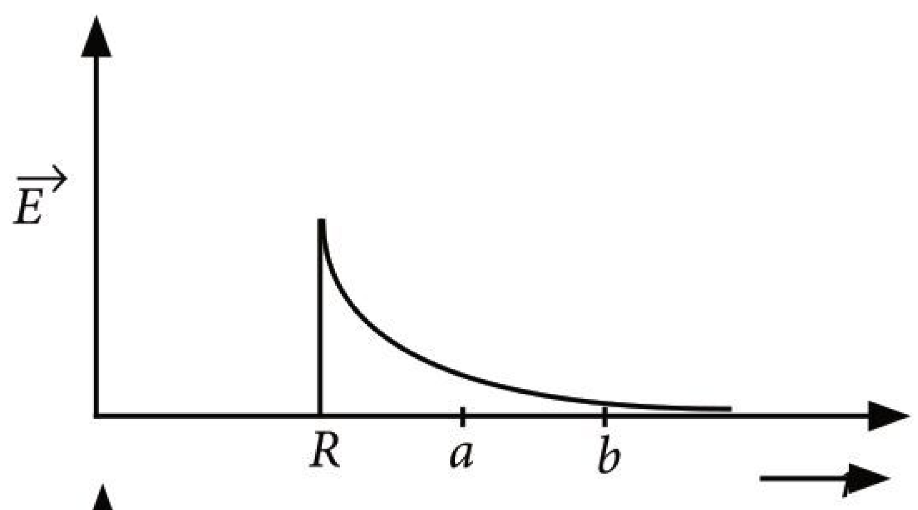
19. The fractional change in the magnetic field intensity at a distance 'r' from centre on the axis of current carrying coil of radius 'a' to the magnetic field intensity at the centre of the same coil is (Take $r < a$).

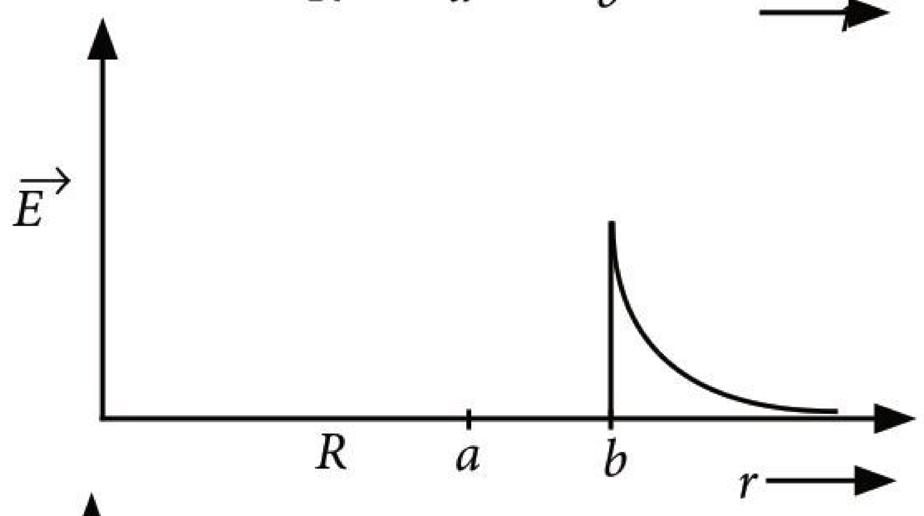
(a) $\dfrac{2}{3}\dfrac{a^2}{r^2}$ (b) $\dfrac{3}{2}\dfrac{a^2}{r^2}$

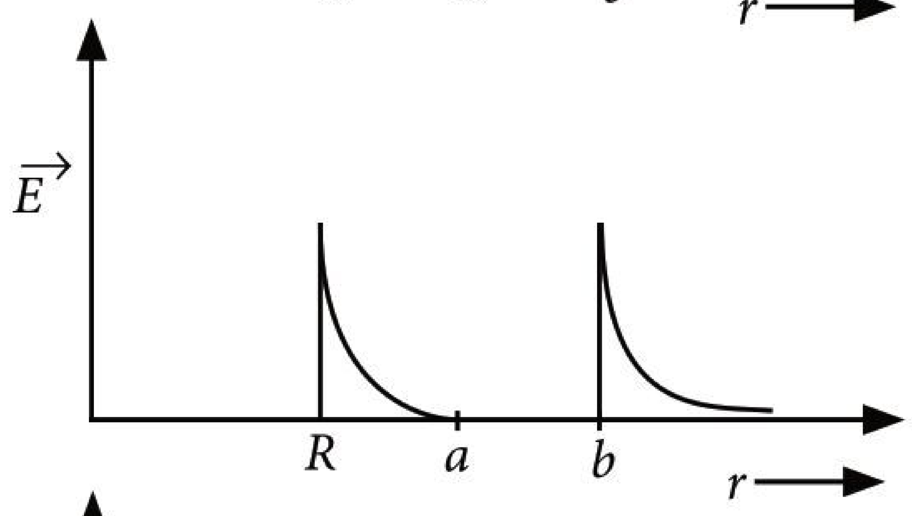
(c) $\dfrac{3}{2}\dfrac{r^2}{a^2}$ (d) $\dfrac{2}{3}\dfrac{r^2}{a^2}$

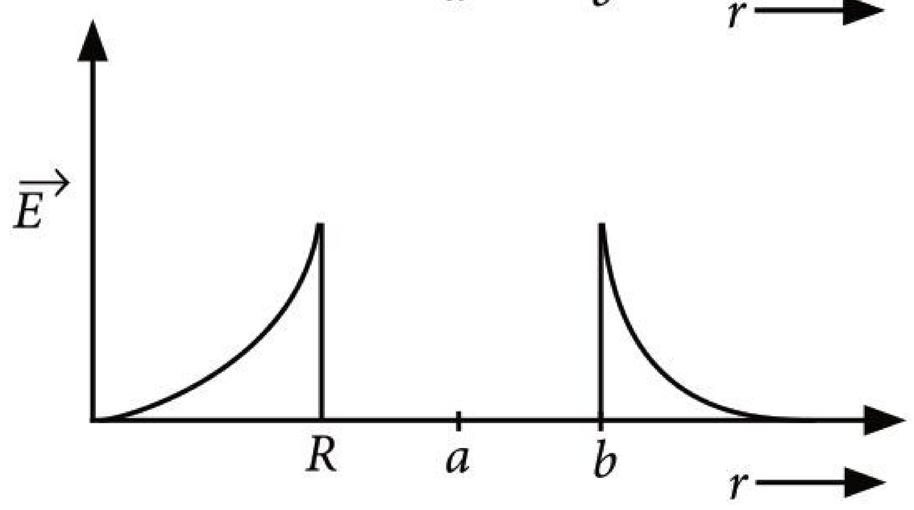
20. A solid metal sphere of radius R having charge q is enclosed inside the concentric spherical shell of inner radius a and outer radius b as shown in the figure. The approximate variation electric field $\vec{E}$ as a function of distance r from centre O is given by



(a)


(b)


(c)


(d)


SECTION-B (NUMERICAL VALUE TYPE)

Attempt any 5 questions out of 10.

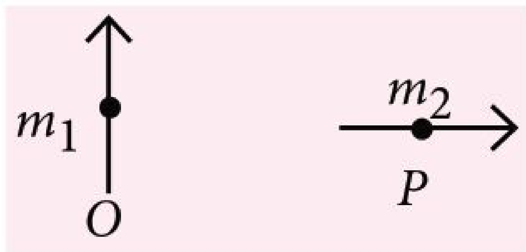
21. An amplitude modulated wave is represented by $C_m(t) = 10(1 + 0.2\cos 12560t)\sin(111 \times 10^4 t)$ volts. The modulating frequency in kHz will be _____.

22. White light is passed through a double slit and interference is observed on a screen 1.5 m away. The separation between the slits is 0.3 mm. The first violet and red fringes are formed 2.0 mm and 3.5 mm away from the central white fringes. The difference in wavelengths of red and violet light is _____ nm.

23. Two travelling waves produces a standing wave represented by equation.
$y = 1.0 \text{ mm} \cos(1.57 \text{ cm}^{-1})x \sin(78.5 \text{ s}^{-1})t.$
The node closest to the origin in the region $x > 0$ will be at $x =$ ____ cm.

24. A uniform chain of length 3 meter and mass 3 kg overhangs a smooth table with 2 meter laying on the table. If k is the kinetic energy of the chain in joules as it completely slips off the table, then the value of k is _____. (Take $g = 10 \text{ m s}^{-2}$)

25. Two short magnetic dipoles m_1 and m_2 each having magnetic moment of 1 A m^2 are placed at point O and P respectively. The distance between OP is 1 meter. The torque experienced by the magnetic dipole m_2 due to the presence of m_1 is _____ $\times 10^{-7}$ N m.



26. A soap bubble of radius 3 cm is formed inside the another soap bubble of radius 6 cm. The radius of an equivalent soap bubble which has the same excess pressure as inside the smaller bubble with respect to the atmospheric pressure is ______ cm.

27. A source and a detector move away from each other in absence of wind with a speed of 20 m s^{-1} with respect to the ground. If the detector detects a frequency of 1800 Hz of the sound coming from the source, then the original frequency of source considering speed of sound in air 340 m s^{-1} will be _____ Hz.

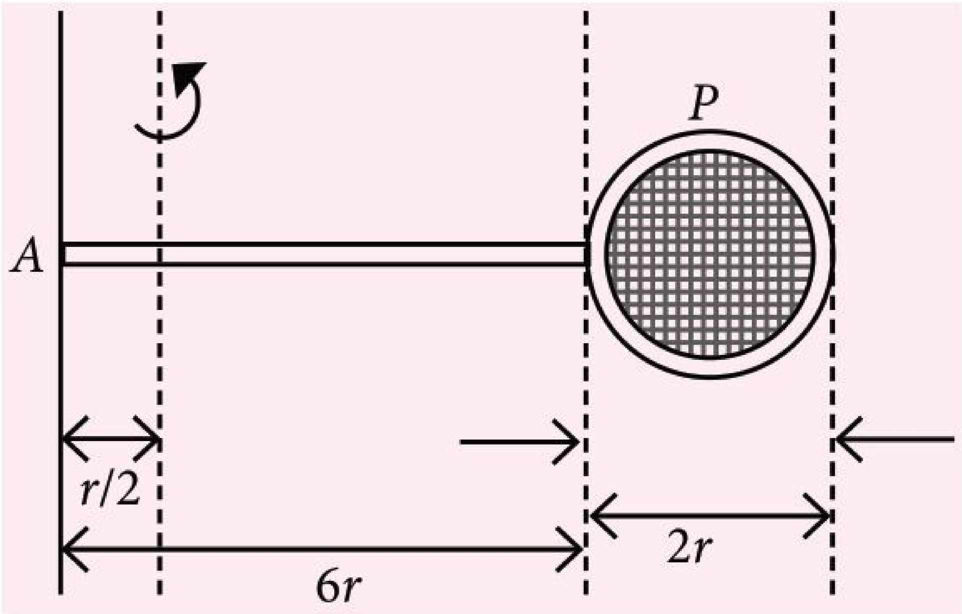
28. The electric field in a plane electromagnetic wave is given by

$$\vec{E} = 200\cos\left[\left(\frac{0.5\times10^3}{m}\right)x - \left(1.5\times10^{11}\,\frac{\text{rad}}{\text{s}}\times t\right)\right]\frac{\text{V}}{\text{m}}\hat{j}$$

If this wave falls normally on a perfectly reflecting surface having an area of 100 cm^2. If the radiation pressure exerted by the electromagnetic wave on the surface during a 10 minute exposure is $\frac{x}{10^9}\frac{\text{N}}{\text{m}^2}$. Find the value of x.

29. two spherical balls having equal masses with radius 5 cm each are thrown upwards along the same vertical direction at an interval of 3 s with the same initial velocity of 35 m s^{-1}, then these balls collide at a height of _____ m. (take $g = 10 \text{ m s}^{-2}$)

30. Consider a badminton racket with length scales as shown in the figure.



If the mass of the linear and circular portions of the badminton racket are same (M) and the mass of the threads are negligible, the moment of inertia of the racket about an axis perpendicular to the handle and in the plane of the ring at, $\frac{r}{2}$ distance from the end A of the handle will be _____ Mr^2.

SOLUTIONS

1. (c) : The root mean square speed of molecules is given by

$$v_{rms} = \sqrt{\frac{3RT}{M}}$$

where, R is gas constant, T is absolute temperature and M is molecular mass.

As the temperature is same, so $v_{rms} \propto \frac{1}{\sqrt{M}}$

Molecular mass of hydrogen < oxygen < CO_2

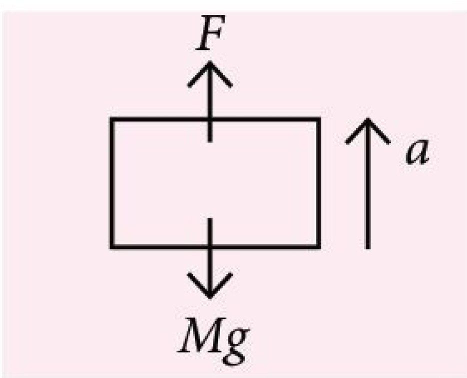
so, $v_H > v_0 > v_C$

2. (b): Initial mass of rocket, M = 100 kg

Acceleration, a = 20 m s^{-1}

Relative speed of gases, u = 500 m s^{-1}

g = = 10 m s^{-1}



Thrust, $F = u \cdot \frac{dm}{dt}$

where, $\frac{dm}{dt}$ is rate of mass burnt

By Newton's second law, $F - mg = ma$

$$u \cdot \frac{dm}{dt} = m(g+a);\ 500\frac{dm}{dt} = 1000(10+20)$$

$$\frac{dm}{dt} = 60 \text{ kg sec}^{-1}$$

3. (b): Frequency, f = 2.92 × 10^{15} Hz

n = 3 to n = 1

Let the frequency is f from n = 2 to n = 1

Also, $\frac{1}{\lambda} = R_H\left(\frac{1}{n_1^2} - \frac{1}{n_2^2}\right)$

So, $f \propto \frac{1}{\lambda}$

So, $Kf = R_H\left(\frac{1}{1^2} - \frac{1}{3^2}\right) = R_H \times \frac{8}{9}$ ...(i)

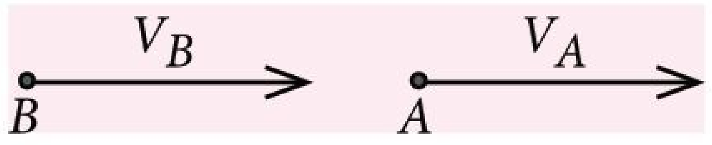
and $Kf' = R_H\left(\frac{1}{1^2} - \frac{1}{2^2}\right) = R_H \times \frac{3}{4}$ ...(ii)

$$\Rightarrow \frac{f'}{f} = \frac{3\times 9}{4\times 8} = \frac{27}{32}$$

$$\Rightarrow f' = 2.92\times 10^{15} \times \frac{27}{32} = 2.46 \times 10^{15} \text{ Hz}$$

4. (a) : Inside a uniform spherical shell, the gravitational field is zero. So, gravitational potential is same everywhere.

5. (a) : Focal length, f = 10 cm; Distance of car B from car A, u = 1.9 m ⇒ u = –190 cm



The mirror is convex in nature, as it is rear view mirror.

$V_{B/A}$ = 40 m s^{-1} = $V_{0/m}$ = velocity of object with respect to mirror.

Velocity of image with respect to mirror is

$V_{1/m} = -m^2\, V_{0/m}$ ...(i)

The magnification of mirror is

$$m = \frac{f}{f-u} = \frac{10}{10+190} = \frac{10}{200} = \frac{1}{20}$$

$$V_{1/m} = -\left(\frac{1}{20}\right)^2 \times 40 = 0.1 \text{ m s}^{-1}$$

6. (d): Dimension of E = [ML^2 T^{-2}]

Dimension of L = [ML^2 T^{-1}]

Dimension of M = [M]

Dimension of G = [M^{-1}L^{3}T^{-2}]

$$\text{Dimension of } P = \frac{EL^2}{M^5G^2} = \frac{[ML^2T^{-2}][ML^2T^{-1}]^2}{[M]^5[M^{-1}L^3T^{-2}]^2} = [M^0L^0T^0]$$

7. (b): Heat/min, $\Delta Q/t$ = 600 J per min

Work, W/s = 90 J s^{-1}

Change in internal energy, ΔU = 2.5 × 10^{3}J

Let the time is t.

Using second law of thermodynamics

$$\frac{\Delta Q}{\Delta t} = \frac{\Delta U}{t} + \frac{\Delta W}{t'} \Rightarrow \frac{6000}{60} = \frac{2.5\times 10^3}{t} + 90$$

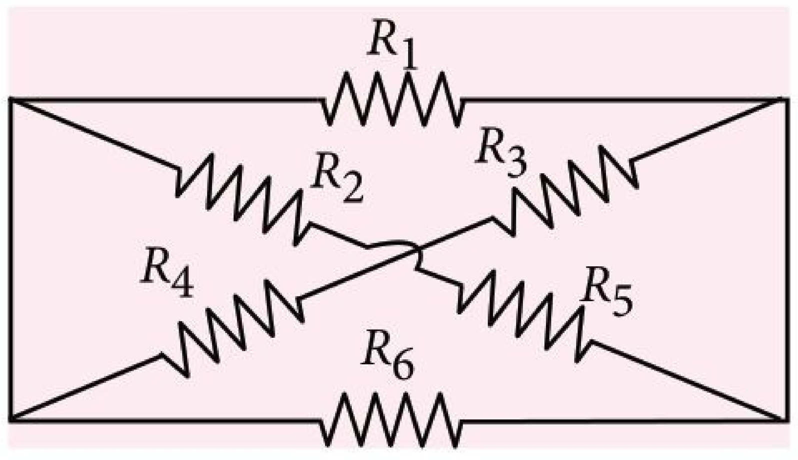
t = 250 s

8. (b): $V = 300$ V, $f = 50$ Hz
$R = 3$ KΩ, $X_L = 250\pi\ \Omega$, $\pi^2 = 10$
For maximum average power, the condition is for resonance.
Let capacitance is C.

$$X_L = X_C \Rightarrow 2\pi f L = \frac{1}{2 \times \pi \times 50 \times C}$$

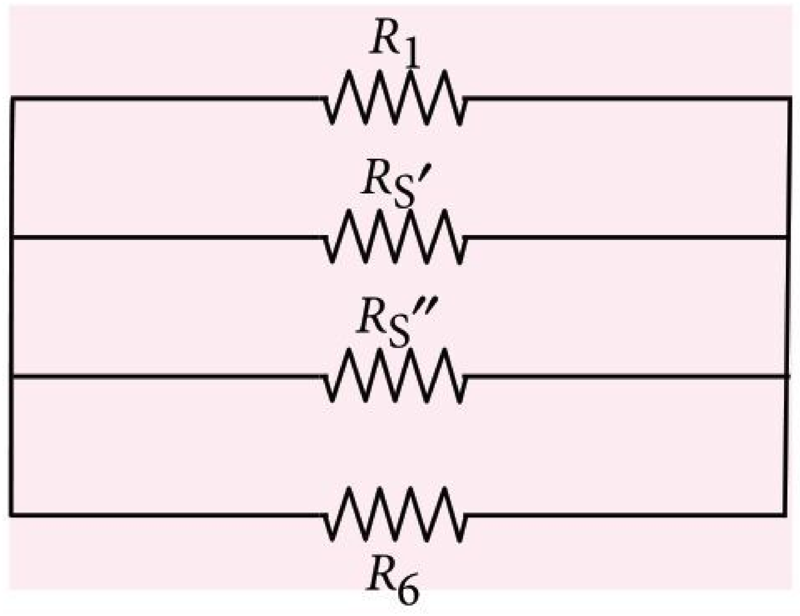
$C = 4 \times 10^{-6}$F $= 4\ \mu$F

9. (d): Here, R_2 and R_5 in series
$R_S' = R_2 + R_5 = 4 + 8 = 12\ \Omega$ and R_4 and R_3 in series
$R_S'' = R_4 + R_3 = 2 + 4 = 6\ \Omega$
So, the circuit is given here.



R_1, R_S', R_S'' and R_6 are in parallel.

$$\frac{1}{Req} = \frac{1}{4} + \frac{1}{8} + \frac{1}{12} + \frac{1}{6}$$



$$\frac{1}{Req} = \frac{6+3+2+4}{24} = \frac{15}{24}$$

$$Req = \frac{24}{15} = 1.6\ \Omega$$

Now, $R = Req + r = 1.6 + 0.6 = 2.2\ \Omega$

$$P = \frac{V^2}{R} = \frac{2.2 \times 2.2}{2.2} = 2.2\ \text{W}$$

10. (d): Incident wavelength, $\lambda = 280$ nm
Work function, $\phi = 2.5$ eV
New incident wavelength, $\lambda' = 400$ nm
$h = 6.63 \times 10^{-34}$ J s, $c = 3 \times 10^8$ m s^{-1}
Let the initial stopping potential is V_0 and the new is V_0'.
By the equation of photo electric effect
$KE_{max} = E - \phi$ where E is incident energy,

$$\Rightarrow \frac{hc}{\lambda} - \phi = eV_0$$

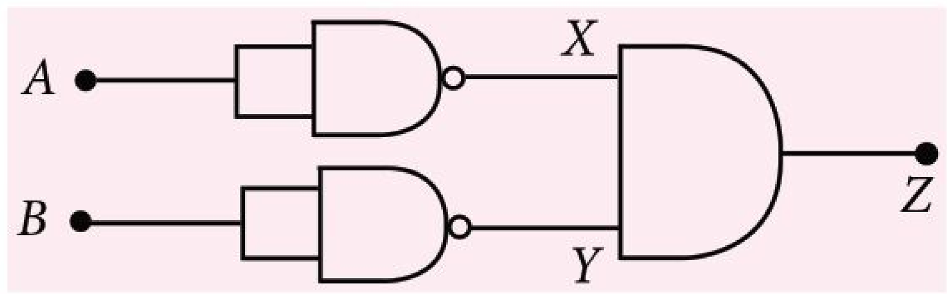
$$eV_0 = \frac{1240}{280} - 2.5 = 1.93\ \text{eV}$$

$V_0 = 1.93$ V
For second case

$$eV_0' = \frac{1240}{400} - 2.5 = 0.6\ \text{eV}$$

$V_0' = 0.6$ V
So, $\Delta V = V_0 - V_0' = 1.93 - 0.6 = 1.33$ V

11. (d):



A	B	X	Y	Z
0	0	1	1	1
0	1	1	0	0
1	0	0	1	0
1	1	0	0	0

It is the truth table of NOR gate.

12. (a) : Diameter, $d = 2$mm
Radius, $r = 1$ mm ; $R_{eq} = 3\ \Omega$
Resistivity of iron, $\rho_{iron} = 12 \times 10^{-8}\Omega$ m
Resistivity of copper-nickel allay, $\rho_{cu} = 51 \times 10^{-8}\Omega$ m
Let l is the length of each of wire.

$$R_1 = \rho_i \frac{l}{A_i}; \ R_2 = \rho_{cu} \frac{l}{A_{cu}}$$

$R_{eq} = 3$

According to question, $\dfrac{R_1 R_2}{R_1 + R_2} = 3$

$$\Rightarrow \frac{\dfrac{12\times10^{-8} \times l}{\pi \times 1 \times 10^{-6}} \times \dfrac{51 \times 10^{-8} \times l}{\pi \times 1 \times 10^{-6}}}{\dfrac{12\times10^{-8} \times l}{\pi \times 1 \times 10^{-6}} + \dfrac{51 \times 10^{-8} \times l}{\pi \times 1 \times 10^{-6}}} = 3$$

So, $l = 97$ m

13. (b): Energy stored in inductor, $U = 64$ J
Current, $I = 8$ A
Power, $P = 640$ W
Let the inductance is L and resistance is R.

$$\text{So, } U = \frac{1}{2}LI^2 \ ; \ 64 = \frac{1}{2} \times L \times 8 \times 8$$

$L = 2$ H
Also $P = I^2R$
$8 \times 8 \times R = 640$; $R = 10\ \Omega$

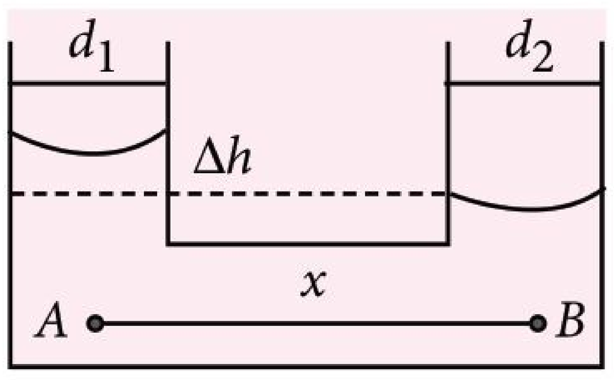
Time constant is given by, $\tau = \dfrac{L}{R} = \dfrac{2}{10} = 0.2$ s

14. (c) : 5^{th} division coincide with reference line, so, it is zero error.
Number of divisions on circular scale, $N = 50$
Pitch = 0.5 mm
Main scale reading, MSR = 5 mm
Circular scale reading, CSR = 20^{th} div.

Least count, L.C. $= \frac{\text{Pitch}}{N} = \frac{0.5}{50} = 0.01$ mm

Reading = MSR + CSR × LC – Zero error × L.C.
= 5 + 0.01 × 20 – 0.5 × 0.01 = 5.15 mm

15. (c) : Diameter, $d_1 = 5$ mm, $d_2 = 8$ mm
Surface tension,
$T = 7.3 \times 10^{-2}$ N m^{-1}
Angle of contact, $\theta = 0°$
$g = 10$ m s^2
Density of water,
$\rho = 10^3$ mg m^{-3}



The pressure at same horizontal level is same, so
$P_A = P_B$

$$P_{atm} - \frac{2T}{r_1} + \rho g(x + \Delta h) = P_{atm} - \frac{2T}{r_2} + \rho g x$$

where, r_1 and r_2 is radius of limbs,
x is the level of P_B and Δh is difference in level of two limbs.

$$\rho g\,\Delta h = 2T\left(\frac{1}{r_1} - \frac{1}{r_2}\right)$$

$$10^3 \times 10 \times \Delta h = 2 \times 7.3 \times 10^{-2}\left[\frac{2}{5 \times 10^{-3}} - \frac{2}{8 \times 10^{-3}}\right]$$

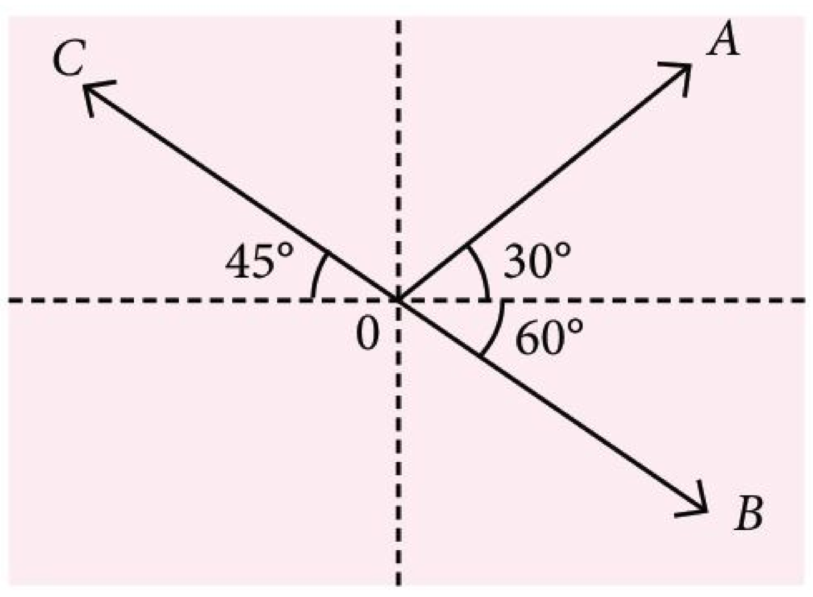
$$10^4 \cdot \Delta h = 2 \times 7.3 \times 10^{-2} \times 10^3 \times 2\left[\frac{1}{5} - \frac{1}{8}\right]$$

$\Delta h = 2.19 \times 10^{-3}$ m = 2.19 mm

16. (b): Let the magnitude of each vector is K.

$$\overrightarrow{OA} = K\left[\cos 30\hat{i} + \sin 30\hat{j}\right]$$

$$\overrightarrow{OA} = K\left[\frac{\sqrt{3}}{2}\hat{i} + \frac{1}{2}\hat{j}\right]$$



$$\overrightarrow{OB} = K\left[\cos 60\,\hat{i} - \sin 60\hat{j}\right] = K\left(\frac{1}{2}\hat{i} - \frac{\sqrt{3}}{2}\hat{j}\right)$$

$$\overrightarrow{OC} = K\left[-\cos 45\,\hat{i} + \sin 45\hat{j}\right] = K\left(-\frac{1}{\sqrt{2}}\hat{i} + \frac{1}{\sqrt{2}}\hat{j}\right)$$

$$\overrightarrow{OA} + \overrightarrow{OB} - \overrightarrow{OC}$$

$$= \left[\frac{\sqrt{3}}{2} + \frac{1}{2} + \frac{1}{\sqrt{2}}\right]\hat{i}\,K + \left[\frac{1}{2} - \frac{\sqrt{3}}{2} - \frac{1}{\sqrt{2}}\right]\hat{j}\,K$$

The direction is given by

$$\tan\theta = \frac{\frac{1}{2} - \frac{\sqrt{3}}{2} - \frac{1}{\sqrt{2}}}{\frac{\sqrt{3}}{2} + \frac{1}{2} + \frac{1}{\sqrt{2}}} = \frac{1 - \sqrt{3} - \sqrt{2}}{1 + \sqrt{3} + \sqrt{2}}$$

$$\theta = \tan^{-1}\frac{(1 - \sqrt{3} - \sqrt{2})}{(1 + \sqrt{3} + \sqrt{2})}$$

17. (a) : Resistivity, $\rho = 200\ \Omega$ m
Capacitance, $C = 2$ pF ; Voltage, $V = 40$ V
Relative permittivity, $\varepsilon_r = 50$
The leakage current is given by

$$i = \frac{q_0 e^{-t/e\varepsilon_r\varepsilon_0}}{\rho\varepsilon_r\varepsilon_0} \ ; \ i = \frac{C \times V e^{-t/e\varepsilon_r\varepsilon_0}}{\rho\varepsilon_r\varepsilon_0}$$

For maximum current.

$$i_{max} = \frac{CV}{\rho\varepsilon_r\varepsilon_0} = \frac{2 \times 10^{-2} \times 40}{200 \times 50 \times 8.85 \times 10^{-12}}$$

$i_{max} = 0.9 \times 10^{-3}$ A = 0.9 mA

18. (a) : In the pentavalent material, the electron density increases, but overall semiconductor is neutral.

19. (c) : The magnetic field on the axis of current carrying coil is given by, $B_{axis} = \dfrac{\mu_0 iR^2}{2(R^2 + x^2)^{\frac{3}{2}}}$

where, i = current, R = Radius of coil, x is distance from centre of coil.
The magnetic field at the centre of coil is

$$B_{Center} = \frac{\mu_0 i}{2a};\ B_{axis} = \frac{\mu_0 ia^2}{2(a^2 + r^2)^{\frac{3}{2}}}$$

Fractional change $= \dfrac{B_c - B_{axis}}{B_c}$

$$= \frac{\frac{\mu_o i}{2a} - \frac{\mu_o i a^2}{2(a^2+r^2)^{\frac{3}{2}}}}{\frac{\mu_o i}{a}} = 1 - \frac{1}{\left(1 + \left(\frac{r^2}{a^2}\right)\right)^{\frac{3}{2}}}$$

$$= 1 - \left[1 + \frac{r^2}{a^2}\right]^{-\frac{3}{2}} = 1 - 1 + \frac{3}{2}\frac{r^2}{a^2} \text{ (Using binomial)}$$

$$= \frac{3}{2}\frac{r^2}{a^2}$$

20. (c) : Assume that the outer spherical shell is non conducting.

The electric field inside a metal sphere is zero.

So, $r < R \Rightarrow E = 0$

And $a > r \geq R \Rightarrow E = \frac{KQ}{r^2} \Rightarrow E \propto \frac{1}{r^2}$

$a \leq r < b \Rightarrow E = 0$

$r \geq b \Rightarrow E = \frac{KQ}{r^2} \Rightarrow E \propto \frac{1}{r^2}$

21. (2) : $C_m(t) = 10\,(1 + 0.2\cos 12560\,t)\sin(111 \times 10^4 t)$ V

Frequency of modulating signal is, $\omega_m = 12560$

$2\pi f = 12560 \Rightarrow f = 2000$ Hz = 2 KHz

22. (300) : Distance of screen, $D = 1.5$ m

Separation between slits, $d = 0.3$ mm

y_r (red) = 3.5 mm, y_i (violet) = 2 mm

Ler λ_r and λ_v be the respective wavelength of red and violet.

The position of bright fringe is, $y = \frac{nD\lambda}{d}$

For red fringes, $y_r = \frac{D\lambda_r}{d} = 3.5$ ...(i)

For violet fringes, $y_v = \frac{D\lambda_v}{d} = 2$ ...(ii)

So, $\lambda_r - \lambda_v = (1.5 \times 10^{-3}) \times \frac{0.3 \times 10^{-3}}{1.5} = 300$ nm

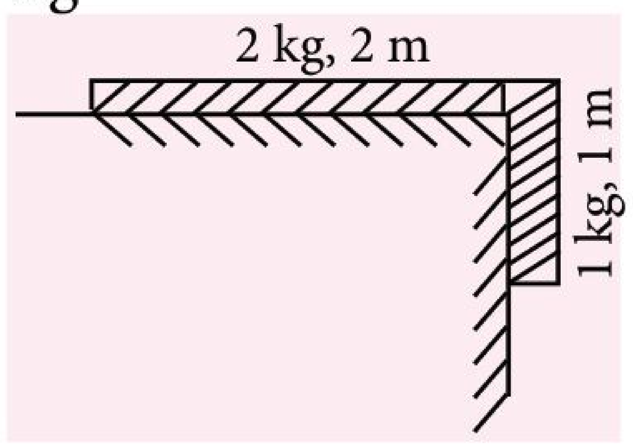
23. (1) : $y = 1$ mm $\cos(1.57 \text{ cm}^{-1})x \sin(78.5 \text{ s}^{-1})t$

For nodes, amplitude is zero.

So $\cos(1.57)\,x = 0 = \cos \pi/2$

$x = \frac{\pi}{2 \times 1.57} = 1$ cm

24. (40) : Mass of chain, $M = 3$ kg



Length of chain, $L = 3$ m

Mass of hanging portion, $m = 1$ kg

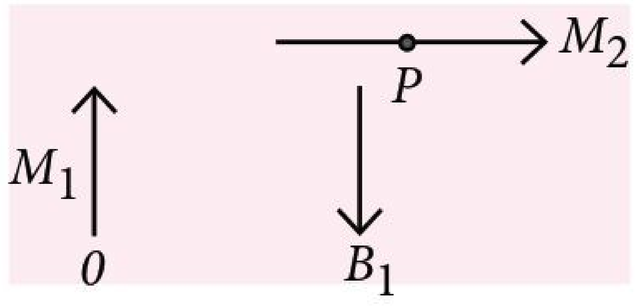
Length of hanging portion, $l = 1$ m

Use conservation of energy,

$0 + (-1 \times 10 \times \frac{1}{2}) = k_f + \left(-3 \times 10 \times \frac{3}{2}\right)$

$-5 = k_f - 45;\ \ k_f = 40$ J

25. (1) : $M_1 = 1$ A m$^2 = M_2$



$OP = 1$m

Torque is given by

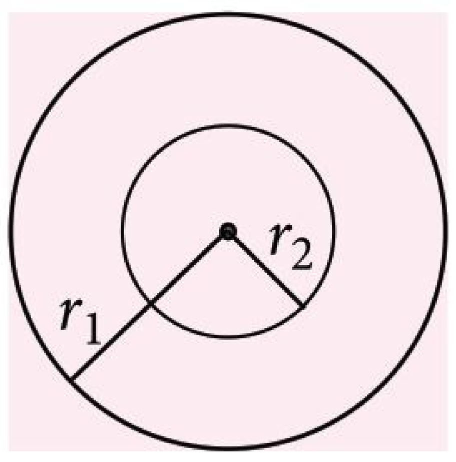
$\vec{\tau} = \vec{M}_2 \times \vec{B}_1$

B_1 is the magnetic field at P due to M_1.

$\tau = M_2 B_1 \sin 90° = 1 \times \frac{\mu_0}{4\pi} \cdot \frac{M_1}{(1)^3} \times 1$

$\tau = 10^{-7}$ N m

26. (2) : $r_2 = 3$ cm = radius of inside bubble



$r_1 = 6$ cm = radius of outside bubble

Let R is the radius of equivalent soap bubble.

Excess pressure inside the smaller soap bubble is

$\Delta P = \frac{4S}{r_1} + \frac{4S}{r_2}$ ...(i)

Excess pressure inside the equivalent soap bubble is

$\Delta P = \frac{4S}{R}$ ...(ii)

where, S is surface tension of soap

From equation (i) and (ii)

$\frac{4S}{R} = \frac{4S}{r_1} + \frac{4S}{r_2}$

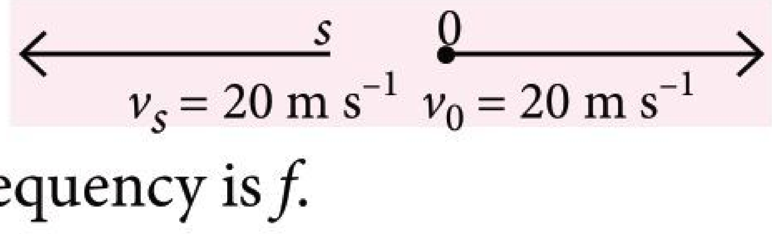
$\frac{1}{R} = \frac{1}{r_1} + \frac{1}{r_2} = \frac{1}{3} + \frac{1}{6} = \frac{2+1}{6} = \frac{1}{2}$

$R = 2$ cm

27. (2025) : Let v is the speed of sound, $v = 340$ m s^{-1}

By the formula of Doppler's effect frequency,

$f' = 1800$ Hz (heard)



Let true frequency is f.

$f' = f\left(\frac{v - v_0}{v + v_s}\right);\ \ 1800 = f\left(\frac{340-20}{340+20}\right)$

$f = 2025$ Hz

28. (354) :

$$\overline{E} = 200\cos\left[\frac{0.5\times10^3}{m}\cdot x - \frac{1.5\times10^{11}\times t}{s}\right]\frac{\text{V}}{m}\hat{j}$$

Area, $A = 100\ \text{cm}^2$

Time, $t = 10$ min

The amplitude of electric field, $E_0 = 200$

Intensity is $I = \frac{1}{2}\in_0 E_0^2 C$

The radiation pressure is

$$P = \frac{2I}{C} = \frac{2}{C}\times\frac{1}{2}\times\in_0 E_0^2 C$$

$$P = \in_0 E_0^2$$

$$P = 8.85\times10^{-12}\times200\times200 = 8.85\times10^{-8}\times4$$

$$P = \frac{354}{10^9}$$

29. (50) : Mass of each ball is m; radius, $r = 5$ cm

Initial velocity, $u = 35\ \text{m s}^{-1}$

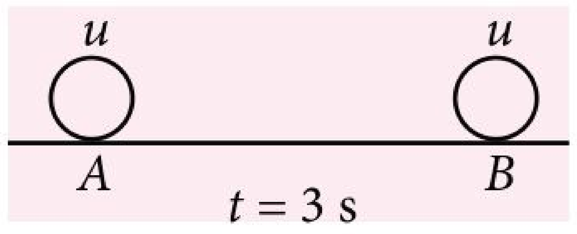
Let both ball collides after time 't' and at height h.

Distance covered by A in time t = distance covered by B in time $(t - 3)$

$$35t - \frac{1}{2}\times10t^2 = 35\,(t-3) - \frac{1}{2}\times10\,(t-3)^2$$

$$35t - \frac{1}{2}\times10t^2 = 35\,t - 105 - \frac{1}{2}\times10\ t^2 - \frac{1}{2}\times10\times9$$

$$+\frac{1}{2}\times10\times6t$$

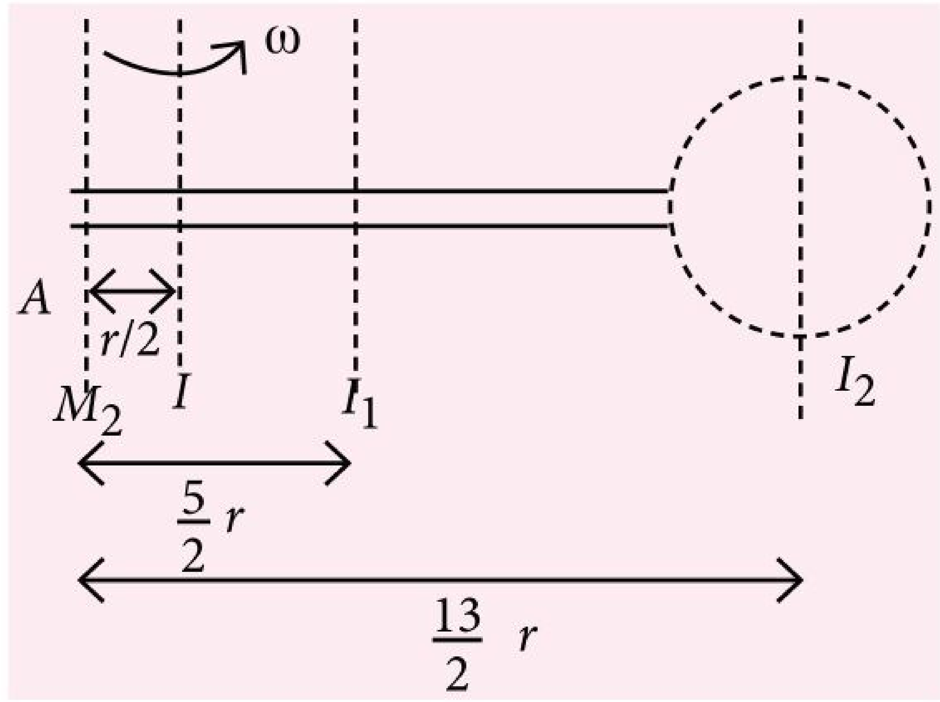


$0 = 150 - 30t \Rightarrow t = 5$ s.

So, height, $h = 35\times5 - \frac{1}{2}\times10\times5\times5$; $h = 50$ m

30. (52) : Mass of linear and circular portion = M

Let I be the MI.



I_1 is about the centre of rod,

I_2 is about centre of circular part.

By using parallel axis theorem.

$$I = \left[I_1 + M\left(\frac{5}{2}r\right)^2\right] + \left[I_2 + M\left(\frac{13r}{2}\right)^2\right]$$

$$I = \left[M\times\frac{36r^2}{12} + M\frac{(25r^2)}{4}\right] + \left[\frac{Mr^2}{2} + \frac{169Mr^2}{4}\right]$$

$I = 52\ Mr^2$

JEE WORKOUTS

Single Option Correct Type

1. A particle of mass m is located in a unidimensional potential field where the potential energy of the particle depends on the coordinates as $U(x) = U_0(1 - \cos ax)$, where U_0 and a are constants. Find the period of small oscillations that the particle performs about the equilibrium position.

(a) $T = \sqrt{\frac{2\pi}{a}\frac{m}{U_0}}$ (b) $T = \frac{2\pi}{a}\sqrt{\frac{m}{U_0}}$

(c) $T = \frac{4\pi}{2a^2}\frac{m}{U_0^2}$ (d) $T = \frac{2\pi^2}{a^2}\frac{m^2}{U_0^2}$

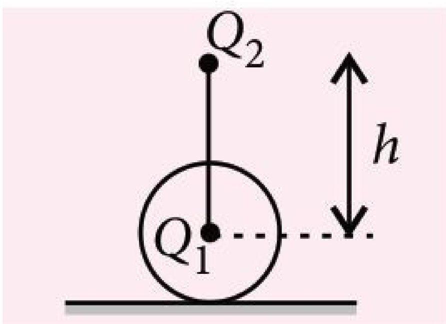
2. An enemy fighter jet is flying at a constant height of 250 m with a velocity of 500 m s^{-1}. The fighter jet passes over an anti-aircraft gun that can fire at any time and in any direction with a speed of 100 m s^{-1}. Determine the time interval in second during which the fighter jet is in danger of being hit by the gun bullets.

(a) $5\sqrt{3}$ (b) $4\sqrt{2}$ (c) $2\sqrt{2}$ (d) $\sqrt{2}$

3. A small ball of mass 2×10^{-3} kg having a charge of 1 μC is suspended by a string of length 0.8 m. Another identical ball having the same charge is kept at the point of suspension. Determine the minimum horizontal velocity which should be imparted to the lower ball so that it can make complete revolution.

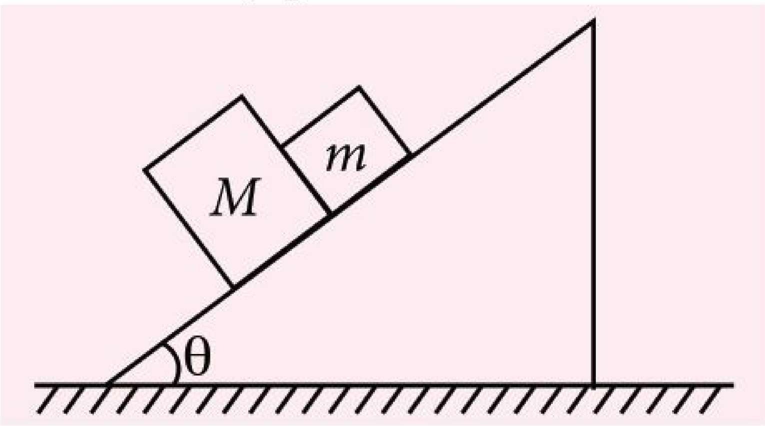
(a) 5.8 m s^{-1} (b) 6.2 m s^{-1}
(c) 4.8 m s^{-1} (d) 3.5 m s^{-1}

4. A point charge $Q_1 = -125$ μC is fixed at the centre of an insulated disc of mass 1 kg. The disc rests on a rough horizontal plane. Another charge $Q_2 = 125$ μC is fixed vertically above the centre of the disc at a height $h = 1$ m. After the disc is displaced slightly in the horizontal direction, find the time period of oscillation of disc.



(a) 2.5 sec (b) 0.6 sec (c) 4.8 sec (d) 1.8 sec

5. Figure shows that two blocks in contact are sliding down an inclined surface of inclination θ = 30°. The friction coefficient between the block of mass m = 2 kg and the incline is μ_1 = 0.20 and that between the block of mass M = 4 kg and the incline is μ_2 = 0.30. Find the acceleration of 2 kg block.



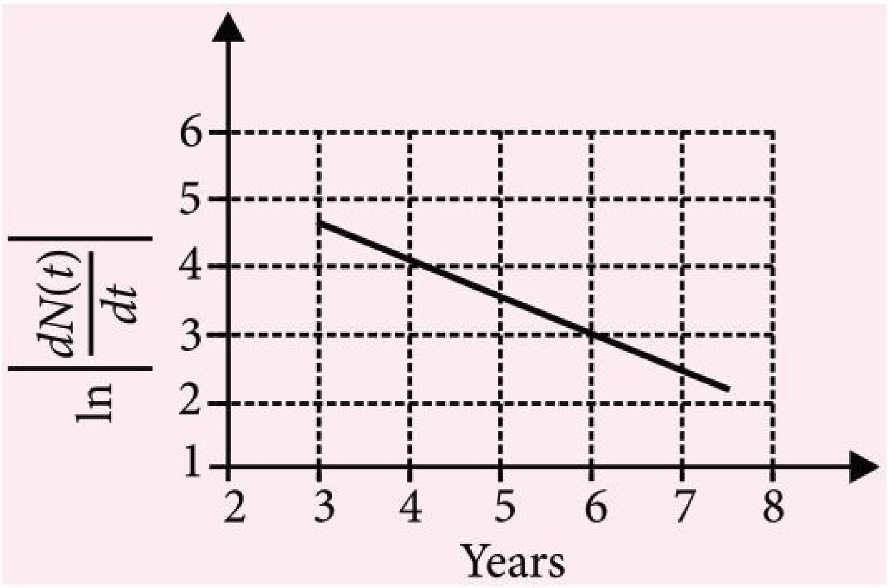
(a) 7.2 m s^{-2} (b) 4.5 m s^{-2}
(c) 3.2 m s^{-2} (d) 2.7 m s^{-2}

Integer Value Type

6. Image of an object approaching a convex mirror of radius of curvature 20 m along its optical axis is observed to move from $\frac{25}{3}$ m to $\frac{50}{7}$ m in 30 seconds. What is the speed of the object in km per hour ?

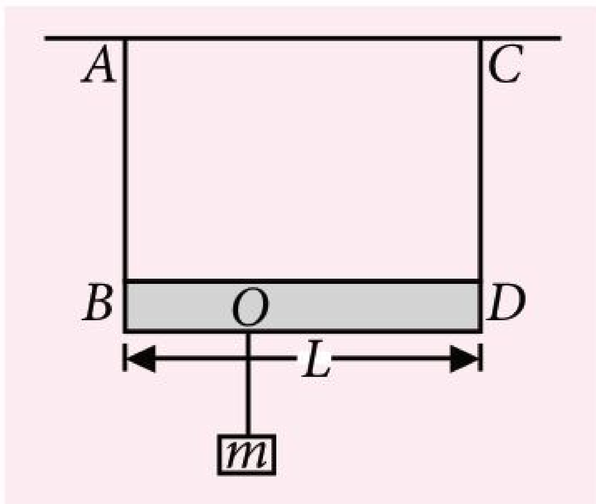
7. To determine the half life of a radioactive element, a student plots a graph of $\ln\left|\frac{dN(t)}{dt}\right|$ versus t. Here $\frac{dN(t)}{dt}$ is the rate of radioactive decay at time t. If the number of radioactive nuclei of this element decreases by a factor of p after 4.16 years, the value

of p is ________.

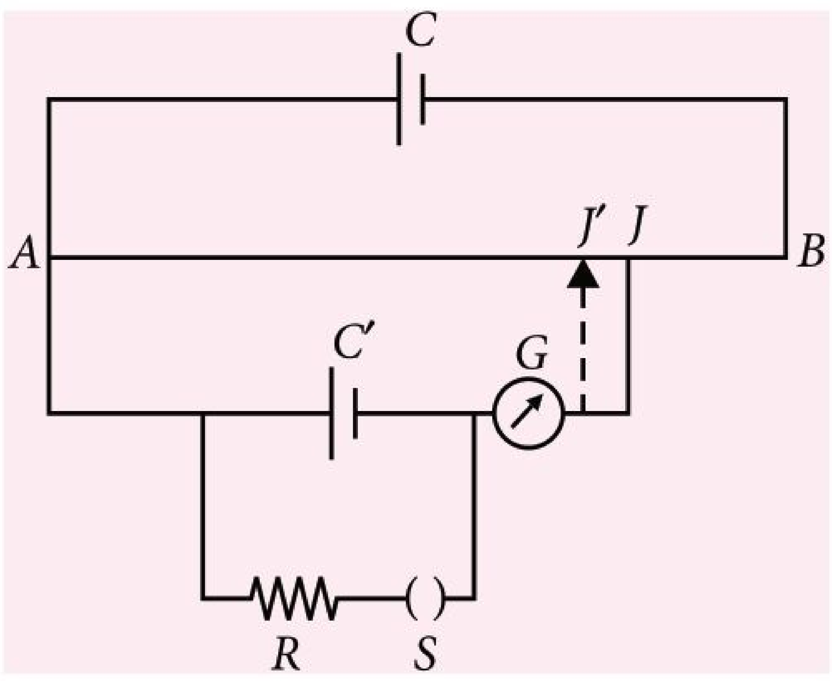


8. A solid copper cube and sphere, both of same mass and emissivity are heated to same initial temperature and kept under identical conditions. The ratio of their initial rate of the fall of temperature is $\left(\frac{p}{\pi}\right)^{1/3}$. Find the value of p.

9. A massless rod of length L is suspended by two identical strings AB and CD of equal lengths. A block of mass m is suspended from point O such that BO is equal to $\frac{L}{n}$. Further it is observed that the frequency of 1st harmonic in AB is equal to 2nd harmonic frequency in CD. Find the value of n.



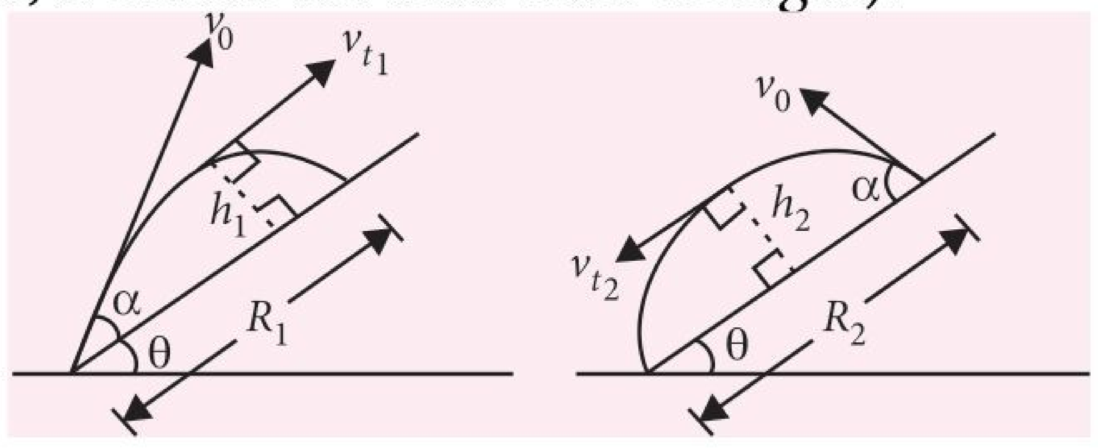
10. Figure shows a potentiometer circuit for determining the internal resistance of a cell. When switch S is open, the balance point is found to be at 76.3 cm of the wire. When switch S is closed and the value of R is 4.0 Ω, the balance point shift to 60.0 cm. Find the internal resistance (in ohm) of cell C'.



One or More Than One Option(s) Correct Type

11. In a photoelectric effect experiment, if υ is the frequency of radiations incident on the metal surface and I is the intensity of the incident radiations, then mark the correct statement(s).
(a) If υ is increased keeping I and work function constant, then stopping potential and maximum kinetic energy of photoelectron increases.
(b) If distance between cathode and anode is changed, then stopping potential remains same.
(c) If I is increased keeping υ and work function constant then stopping potential remains same and saturation current increases.
(d) Work function is decreased keeping υ and I constant then stopping potential increases and maximum kinetic energy of photoelectron increases.

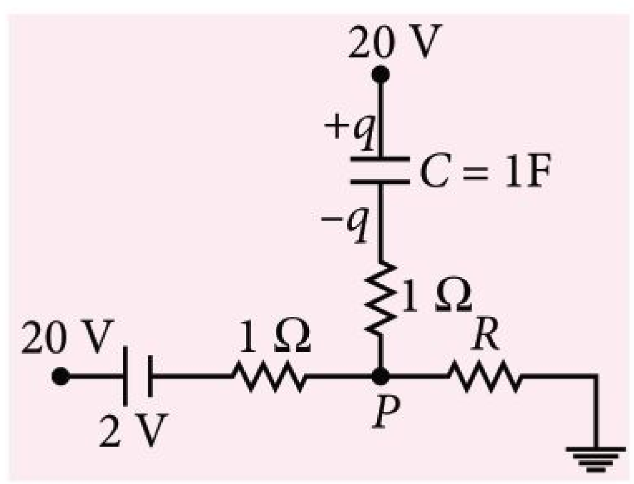
12. Two balls are thrown from an inclined plane at angle of projection α with the plane, one up the incline and other down the incline as shown in figure. (Here, T stands for total time of flight).



Which of the following is/are corrects?
(a) $h_1 = h_2 = \frac{v_0^2 \sin^2 \alpha}{2g\cos\theta}$
(b) $T_1 = T_2 = \frac{2v_0 \sin\alpha}{g\cos\theta}$
(c) $R_2 - R_1 = g(\sin\theta)\, T_1^2$
(d) $v_{t_1} = v_{t_2}$

13. Water is flowing smoothly through a closed pipe system. At one point A, the speed of the water is 3.0 m s^{-1} while at another point B, 1.0 m higher, the speed is 4.0 m s^{-1}. The pressure at A is 20 kPa when the water is flowing and 18 kPa when the water flow stops. Then
(a) the pressure at B when water is flowing is 6.7 kPa.
(b) the pressure at B when water is flowing is 8.2 kPa.
(c) the pressure at B when water stops flowing is 10.2 kPa.
(d) the pressure at B when water stops flowing is 8.2 kPa.

14. Figure shows a part of complete circuit. At $t = 0$, charge on capacitor as a function of time is given by $q = 3(1 - e^{-t})$ coulomb. At $t = 0$, mark the correct



statement(s).

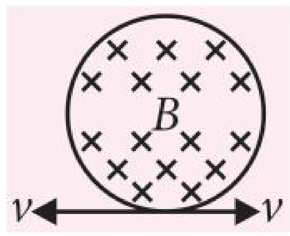
(a) Current through branch containing capacitor is 3 A.

(b) Current through R is 7 A.

(c) The potential at P is 14 V.

(d) The value of R is 2 Ω.

15. A circular conducting loop of radius r_0 and having resistance per unit length λ as shown in the figure is placed in a magnetic field B which is constant in space and time. The ends of the loop are crossed and pulled in opposite directions with a velocity v such that the loop always remains circular and the radius of the loop goes on decreasing, then



(a) radius of the loop changes with t as $r_0 - \frac{vt}{\pi}$

(b) emf induced in the loop as a function of time is $2Bv\left[r_0 - \frac{vt}{\pi}\right]$

(c) current induced in the loop is $\frac{Bv}{2\pi\lambda}$

(d) current induced in the loop is $\frac{Bv}{\pi\lambda}$.

SOLUTIONS

1. **(b):** $U = U_0(1 - \cos ax)$

This is zero when $\cos ax = 1$ or $ax = 0$ or $x = 0$

$\therefore$ $x = 0$ is mean position of system.

At extreme end, potential energy becomes maximum, *i.e.*, $2U_0$

$\therefore$ $2U_0 = U_0(1 - \cos Aa)$ where A is amplitude.

$2 = 1 - \cos Aa$

Since A is small, $\cos aA = 1 - \frac{(Aa)^2}{2}$

$\therefore$ $1 - \left(1 - \frac{(Aa)^2}{2}\right) = 2$ or $A = 2/a$

As maximum kinetic energy $= 2U_0$

$$\therefore \quad 2U_0 = \frac{1}{2}m\omega^2 A^2 = \frac{1}{2}m\omega^2 \frac{4}{a^2}$$

$$\text{or} \quad \omega^2 = \frac{U_0 a^2}{m} \text{ or } \omega = a\sqrt{\frac{U_0}{m}} \Rightarrow T = \frac{2\pi}{a}\sqrt{\frac{m}{U_0}}$$

2. **(c) :** The equation of trajectory of bullet is

$$y = x\tan\theta - \frac{\frac{1}{2}gx^2}{u^2}\sec^2\theta$$

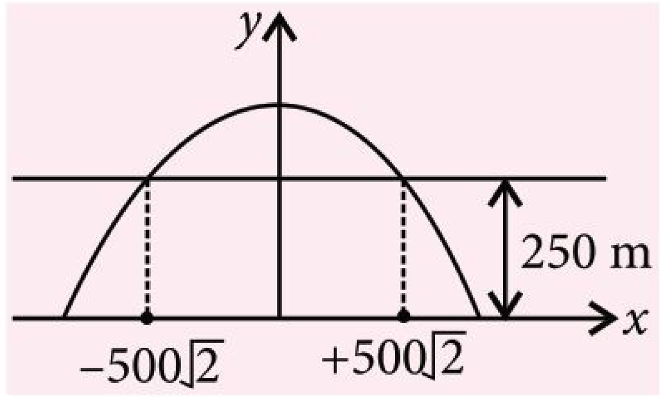
$$= x\tan\theta - \frac{\frac{1}{2}gx^2}{u^2}(1+\tan^2\theta) \quad \text{...(i)}$$

For a given value of x, maximum y can be determined from

$$\frac{dy}{d(\tan\theta)} = x - \frac{\frac{1}{2}gx^2}{u^2}(2\tan\theta) = 0 \text{ or } \tan\theta = \frac{u^2}{gx} \quad \text{...(ii)}$$

Putting (ii) in (i), we get,

$$y_{\max} = \frac{u^2}{2g} - \frac{\frac{1}{2}gx^2}{u^2}$$



The bullet can hit an area defined by

$$y \le \frac{u^2}{2g} - \frac{\frac{1}{2}gx^2}{u^2}$$

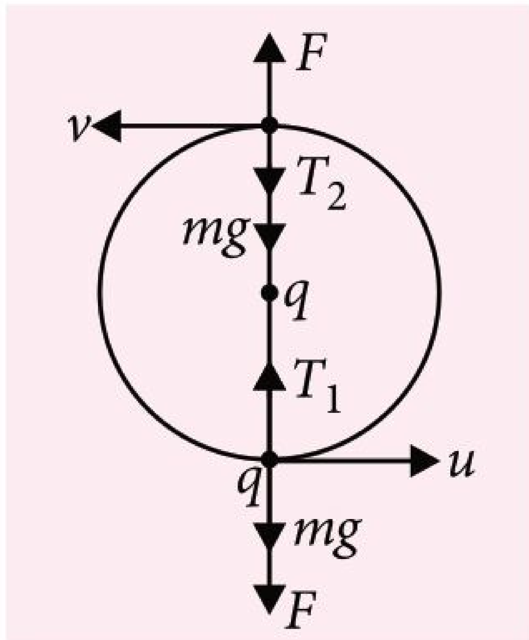
On substituting given values, we get

$$\frac{x^2}{2000} \le 250 \text{ or } -500\sqrt{2} \le x \le +500\sqrt{2}$$

The fighter jet can travel $1000\sqrt{2}$ m while it can be hit. So the plane is in danger for a period of $\frac{1000\sqrt{2}}{500} = 2\sqrt{2}$ s

3. **(a) :** If the ball has to just complete the circle then the tension must vanish at the topmost point, *i.e.* $T_2 = 0$.

From Newton's 2nd law



$$T_2 + mg - \frac{q^2}{4\pi\varepsilon_0 l^2} = \frac{mv^2}{l} \quad \text{...(i)}$$

As $T_2 = 0$, then

$$mg - \frac{q^2}{4\pi\varepsilon_0 l^2} = \frac{mv^2}{l} \quad \text{...(ii)}$$

Using conservation of energy

Energy at topmost point = Energy at lowest point

$$\frac{1}{2}mv^2 + mg(2l) = \frac{1}{2}mu^2$$

$v^2 = u^2 - 4gl$ ...(iii)

Put (iii) in (ii), we get

$$u = \sqrt{5gl - \frac{q^2}{4\pi\varepsilon_0 ml}}$$

Put the given values, we get $u = 5.8$ m s^{-1}

4. **(b)**

5. **(d):** From free body diagram of mass m,

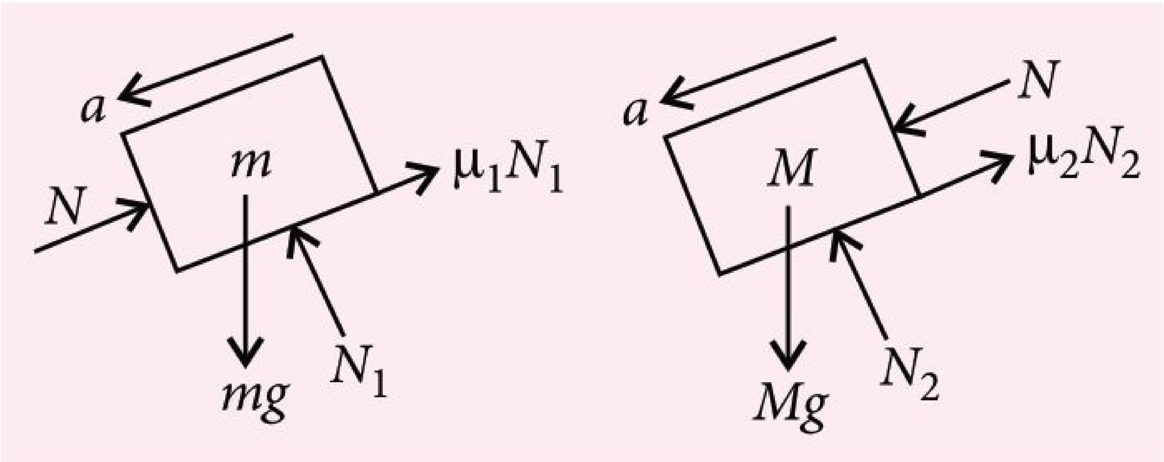
$N_1 = mg\cos\theta$ ...(i)

$ma = mg\sin\theta - \mu_1 N_1 - N$ ...(ii)

From free body diagram of mass M,

$N_2 = Mg\cos\theta$ ...(iii)

$Ma = Mg\sin\theta + N - \mu_2 N_2$ ...(iv)



Putting values of N_1 and N_2 and adding equations (ii) and (iv), we get

$(M + m)a = (M + m)g \sin\theta - (\mu_1 m + \mu_2 M)g\cos\theta$

$$\Rightarrow \quad a = \frac{(M+m)g\sin\theta - (\mu_1 m + \mu_2 M)g\cos\theta}{M+m}$$

Here, $M = 4$ kg, $m = 2$ kg, $\theta = 30°$, $\mu_1 = 0.20$, $\mu_2 = 0.30$, $g = 10 \text{ m s}^{-2}$

$\therefore \quad a = 2.7 \text{ m s}^{-2}$

6. **(3):** Focal length of a convex mirror,

$$f = \frac{R}{2} = \frac{20}{2} \text{ m} = 10 \text{ m}$$

For first object $v_1 = +\frac{25}{3}$ m, $f = +10$ m

Using mirror formula $\frac{1}{v} + \frac{1}{u} = \frac{1}{f}$

$$\frac{1}{u_1} = \frac{1}{10} - \frac{3}{25}\ ;\ u_1 = -50 \text{ m}$$

For second object $v_2 = +\frac{50}{7}$ m, $f = +10$ m

$$\therefore \quad \frac{1}{v_2} + \frac{1}{u_2} = \frac{1}{f}\ ;\ u_2 = -25 \text{ m}$$

Speed of the object $= \frac{25}{30} \text{ m s}^{-1} = 3 \text{ km h}^{-1}$

7. **(8):** According to radioactive decay

$$N = N_0 e^{-\lambda t}\ ;\ \frac{dN}{dt} = -\lambda N_0 e^{-\lambda t}$$

$$\left|\frac{dN}{dt}\right| = \left|-\lambda N_0 e^{-\lambda t}\right| \quad \Rightarrow \left|\frac{dN}{dt}\right| = \lambda N_0 e^{-\lambda t}$$

Taking natural logarithms of both sides of above equation, we get

$$\ln\left|\frac{dN}{dt}\right| = -\lambda t + \ln(\lambda N_0)$$

Comparing the above equation with equation of a straight line *i.e.* $y = mx + c$, we get

From graph, $\text{slope} = -\lambda = \frac{3-4}{6-4} = \frac{1}{2} \text{ year}^{-1}$

Half life $T_{1/2} = \frac{0.693}{\lambda} = 2 \times 0.693 \text{ years} = 1.386$ years

4.16 years is approximately 3 half lives

Nuclei will decay by a factor of $2^3 = 8$

$\therefore \quad p = 8$

8. **(6):** $\frac{dT}{dt} = -\frac{\sigma e A}{ms}(T^4 - T_0^4)$. Here e is same, m is same, both are of copper and both are heated upto same temperature and kept under same conditions, so $(T^4 - T_0^4)$ is same initially. But $(T^4 - T_0^4)$ will not remain same for both every time.

$$\therefore \quad \frac{dT}{dt} \propto -A$$

$$\frac{\left(\frac{dT}{dt}\right)_{\text{cube}}}{\left(\frac{dT}{dt}\right)_{\text{sphere}}} = \frac{A_{\text{cube}}}{A_{\text{sphere}}}$$

For sphere, $\rho \times \frac{4}{3}\pi R^3 = M$; $R = \left(\frac{3M}{4\pi\rho}\right)^{1/3}$,

$$\therefore \quad A_{\text{sphere}} = 4\pi R^2 = 4\pi\left(\frac{3M}{4\pi\rho}\right)^{2/3}$$

For cube, $\rho L^3 = M$; $L = \left(\frac{M}{\rho}\right)^{1/3}$,

$$A_{\text{cube}} = 6L^2 = 6\left(\frac{M}{\rho}\right)^{2/3}$$

$$\frac{\left(\frac{dT}{dt}\right)_{\text{cube}}}{\left(\frac{dT}{dt}\right)_{\text{sphere}}} = \frac{A_{\text{cube}}}{A_{\text{sphere}}} = \frac{6\times\left(\frac{M}{\rho}\right)^{2/3}}{4\pi\left(\frac{3M}{4\pi\rho}\right)^{2/3}} = \left(\frac{6}{\pi}\right)^{1/3}$$

$\therefore \quad p = 6$

9. **(5):** Frequency of first harmonic in AB = Frequency of second harmonic in CD

$$\therefore \quad \frac{1}{2l}\sqrt{\frac{T_1}{\mu}} = \frac{1}{l}\sqrt{\frac{T_2}{\mu}}$$

or $\quad T_1 = 4T_2$ ...(i)

For translational equilibrium,

$T_1 + T_2 = mg$ ...(ii)

From (i) and (ii), we get

$T_1 = \frac{4mg}{5}$ and $T_2 = \frac{mg}{5}$

For rotational equilibrium about O,

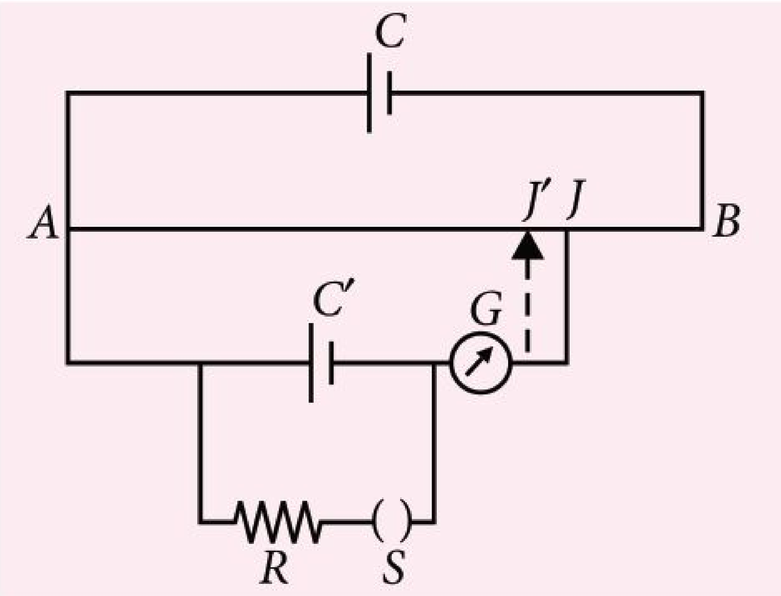
Torque due to T_1 about O = Torque due to T_2 about O

$$T_1\frac{L}{n}=T_2\left(L-\frac{L}{4}\right)$$

or $\frac{4mg}{5n}=\frac{mg}{5}\left(1-\frac{1}{n}\right)$

or $\frac{4}{n}=1-\frac{1}{n}$ or $\frac{5}{n}=1$ or $n=5$

10. (1):



Let ε be the emf of the cell C' and r its internal resistance. Let $l = AJ$ be the balance length when switch S is open. When a resistance R is introduced by closing the switch a current begins of flow through the cell C' and resistance R. The potential difference between the terminals of the cells falls and the balance length decreases to $l' = AJ'$. The terminal resistance of the cell is given by

$$r=\frac{E-V}{I}$$

where V is the terminal voltage of C' and I is the current in the circuit involving C' and R.

Also $I=\frac{V}{R}$. $\therefore\ r=\left(\frac{E}{V}-1\right)R$

But $\frac{E}{V}=\frac{l}{l'}$.

$$\therefore\quad r=R\left(\frac{l-l'}{l'}\right)=4.0\times\left(\frac{76.3-60.0}{60.0}\right)\simeq 1\ \Omega$$

11. (a, b, c, d)

12. (a, b, c) : Maximum height of projectile on an inclined plane, $h_{1\max}=\frac{(v_0\sin\alpha)^2}{2g\cos\theta}=h_{2\max}$

⇒ (a) is correct

Time of flight

$$T_1=\frac{2v_0\sin\alpha}{g\cos\theta}=T_2 \Rightarrow \text{(b) is correct}$$

where α = angle of projection from inclined plane

θ = angle of inclination of surface.

$$R_1=(v_0\cos\alpha)T_1-\frac{1}{2}g\sin\theta T_1^2$$

(Range upward the inclined plane)

$$R_2=(v_0\cos\alpha)T_2+\frac{1}{2}g\sin\theta T_2^2$$

(Range downward the inclined plane)

⇒ $(R_2 - R_1) = g\sin\theta\, T_1^2 \Rightarrow$ (c) is correct

v_{t_1} and v_{t_2} are the velocities of the particles at their maximum height. Let the particles reach their maximum heights at time t_1 and t_2 respectively.

Hence, $0 = (v_0 \sin\alpha) - (g\cos\theta)\, t_1$

$$\Rightarrow\quad t_1=\frac{v_0\sin\alpha}{g\cos\theta}$$

Similarly, $t_2=\frac{v_0\sin\alpha}{g\cos\theta}$.

Hence, $t_2 = t_1$

Hence, $v_{t_1} = v_0\cos\alpha - (g\sin\theta)\, t_1$

$v_{t_2} = v_0\cos\alpha + (g\sin\theta)\, t_2$

$\Rightarrow\quad v_{t_1} \neq v_{t_2}$

13. (a, d) : Let P_1, h_1, and v_1 and P_2, h_2 and v_2 represent the pressures, heights and velocities of flow at the two points respectively. According to the Bernoulli's theorem

$$P_1+\rho g h_1+\frac{1}{2}\rho v_1^2=P_2+\rho g h_2+\frac{1}{2}\rho v_2^2 \qquad ...(i)$$

Putting $v_1 = 3.0\ \text{m s}^{-1}$, $v_2 = 4.0\ \text{m s}^{-1}$, $(h_2 - h_1) = 1$ m, $P_1 = 20$ kPa

we get,

$$P_2=20+\left[10^3\times 9.8\,(-1)+\frac{10^3}{2}[9-16]\right]\times 10^{-3}$$

$= 20 - 9.8 - 3.5 = 6.7$ kPa

Also when the flow stops, $v_1 = v_2 = 0$ and then from (i),

$P_2 = 18 - 9.8 = 8.2$ kPa

14. (a, b, c, d)

15. (a, b, d): Perimeter is decreasing at a rate of $2v$

$$\therefore\quad \frac{d}{dt}(2\pi r)=2v$$

$$\Rightarrow\quad \frac{dr}{dt}=\frac{v}{\pi}\qquad \therefore\ r=(r_0-\frac{v}{\pi}t)$$

$$\Rightarrow\quad \varepsilon=\left|\frac{-d\phi}{dt}\right|=B\cdot 2\pi\cdot r\frac{dr}{dt}\qquad (\because\ \phi=B\cdot\pi r^2)$$

$$\therefore\quad \varepsilon=2B\pi\left(r_0-\frac{v}{\pi}t\right)\frac{v}{\pi}=2Bv\left(r_0-\frac{v}{\pi}t\right)$$

$$I=\frac{\varepsilon}{R}=\frac{2Bvr}{\lambda.2\pi r}=\frac{Bv}{\pi\lambda}$$

CLASS-XI

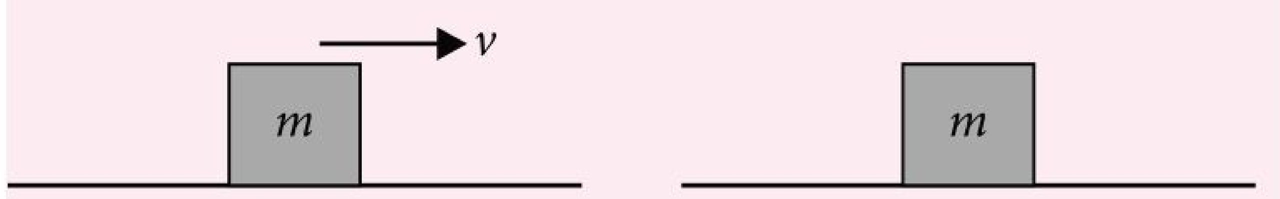
BRUSH UP for NEET/JEE

Brush up your concepts to get high rank in NEET/JEE (Main and Advanced) by reading this column. This specially designed column is updated year after year by a panel of highly qualified teaching experts well-tuned to the requirements of these Entrance Tests.

Unit 3 Laws of Motion | Work, Energy and Power

Newton's First Law of Motion

- A body continues to be in its state of rest or uniform motion along a straight line, unless it is acted upon by some external force to change the state.



State of uniform motion

State of rest (Special case of uniform motion with zero speed)

- Inertia is the property of the body to which body opposes the change of its state.
- Quantitatively, inertia of a body $\propto$ mass of the body.
- **Different types of inertia**

Inertia of rest	Inertia of motion	Inertia of direction
It is the inability of a body to change its state of rest.	It is the inability of a body to change its state of uniform motion.	It is the inability of a body to change its direction of motion.
When we shake a branch of a mango tree, the mangoes fall down.	When a bus or train stops suddenly, passengers sitting inside it tends to fall forward.	Rotating wheels of the vehicle throw out mud, mudguards over the wheels stop this mud.

Newton's Second Law of Motion

- The rate of change of linear momentum of a body is directly proportional to the applied force and takes place in the direction in which the force acts.
- Mathematically, $\vec{F} = \frac{d\vec{p}}{dt}$
- It can be shown experimentally that the acceleration of an object is
  - proportional to the net force $\vec{F}_{net}$ on the object and
  - inversely proportional to the mass m of the object.

  $a \propto F_{net}$ and $a \propto \frac{1}{m}$.

  Thus, $a \propto \frac{F_{net}}{m} \Rightarrow a = \frac{kF_{net}}{m}$,

  The value of constant k is 1 in SI unit.

  $\therefore\ a = \frac{F_{net}}{m}$, $F_{net} = ma$

Forces are Balanced

↓

$a = 0\ \mathrm{m\,s^{-2}}$

Object at rest ($v = 0\ \mathrm{m\,s^{-1}}$)	Object in Motion ($v \neq 0\ \mathrm{m\,s^{-1}}$)
⇓ Stays at rest	⇓ Stays in Motion (Same speed and direction)

Forces are unbalanced

↓

There is an acceleration.

The acceleration depends directly upon the net force.	The acceleration depends inversely upon the mass of the object.

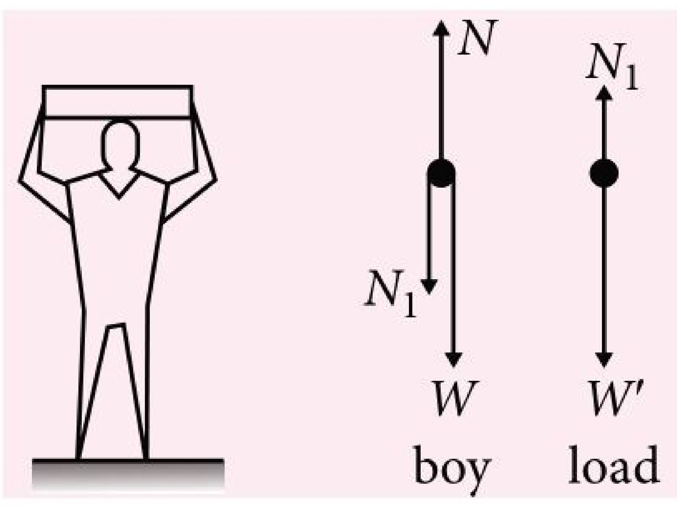
- More fundamental relationship

$$F_{\text{net external}} = \frac{d(mv)}{dt} = m\frac{dv}{dt} + v\frac{dm}{dt}$$

Newton's Third Law of Motion

- It states that to every action, there is an equal (in magnitude) and opposite (in direction) reaction.
- If a body A exerts a force $\vec{F}$ on another body B then B exerts a force $-\vec{F}$ on A. The two forces act along the same line.
- Action and reaction never cancel each other *i.e.*, they cannot balance each other as they act on different bodies.
- This law is applicable whether the bodies are at rest or in motion.

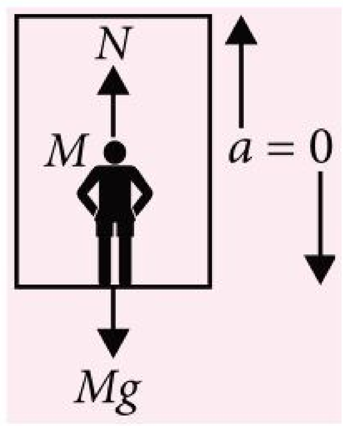
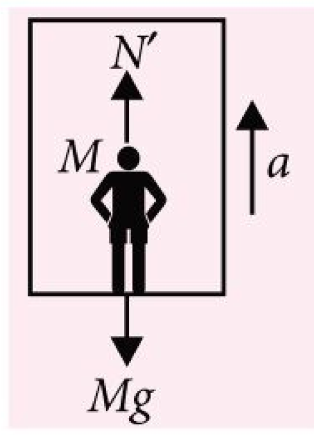
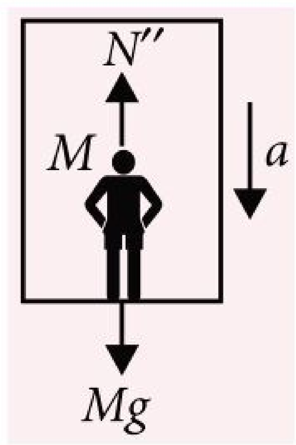
Free Body Diagrams



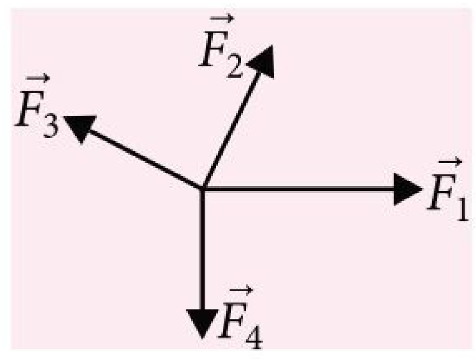
- Body is represented by a dot and each external force is represented by a vector with its tails on the dot.
- Consider a system of a boy and a heavy load. The boy stands on the floor balancing a heavy load on his head.
- List of forces on boy and load

System	Force exerted by	Magnitude of the force	Direction of the force	Nature of the force
Boy	Earth Floor Load	W N N_1	Downward Upward Downward	Gravitational Electromagnetic Electromagnetic
Load	Earth Boy	W' N_1	Downward Upward	Gravitational Electromagnetic

Apparent Weight of a Body in Moving Lift

Vertically upward or downward with uniform velocity ($a = 0$)	Vertically upward with uniform acceleration a	Vertically downward with uniform acceleration a
		
$N = Mg$ = Actual weight of body	$N' = M(g + a)$ $\Rightarrow N' > N$	$N'' = M(g - a)$ $\Rightarrow N'' < N$

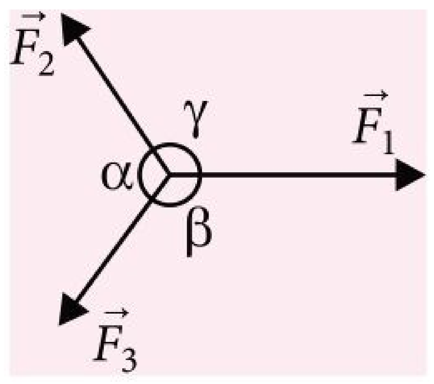
Equilibrium of Forces



- **Concurrent coplanar forces :** If all forces are in equilibrium, or $\vec{F}_1 + \vec{F}_2 + \vec{F}_3 + \vec{F}_4 = 0$, then we can write,

  $\Sigma F_x = 0$ and $\Sigma F_y = 0$

  where x and y are any two mutually perpendicular directions.

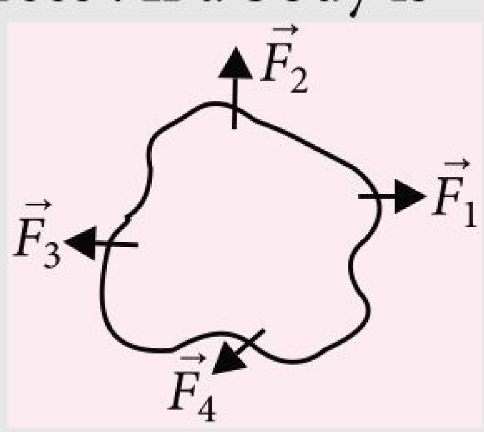


- **Lami's Theorem :** If a body is in equilibrium under three concurrent forces as shown in figure. Then we can write,

$$\frac{F_1}{\sin\alpha} = \frac{F_2}{\sin\beta} = \frac{F_3}{\sin\gamma}$$

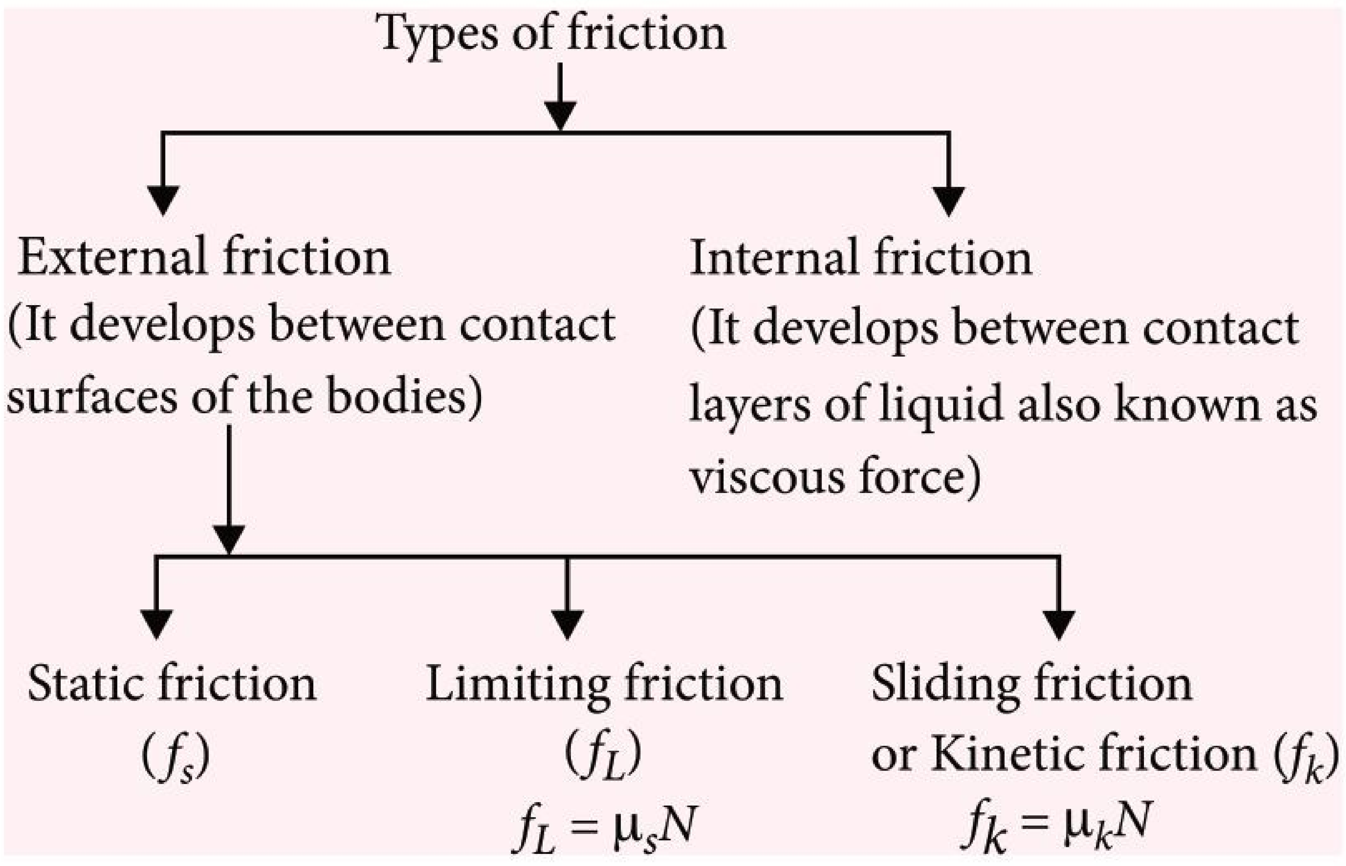
Key Point

- The forces of action and reaction may appear due to actual physical contact of the two bodies or even from a distance.
- Single isolated force is not possible, force is always possible in action reaction pair form.
- Non concurrent coplanar forces : If a body is in equilibrium under non concurrent coplanar forces we can write, $\Sigma F_x = 0$, $\Sigma F_y = 0$ and Σ(moment about any point) = 0

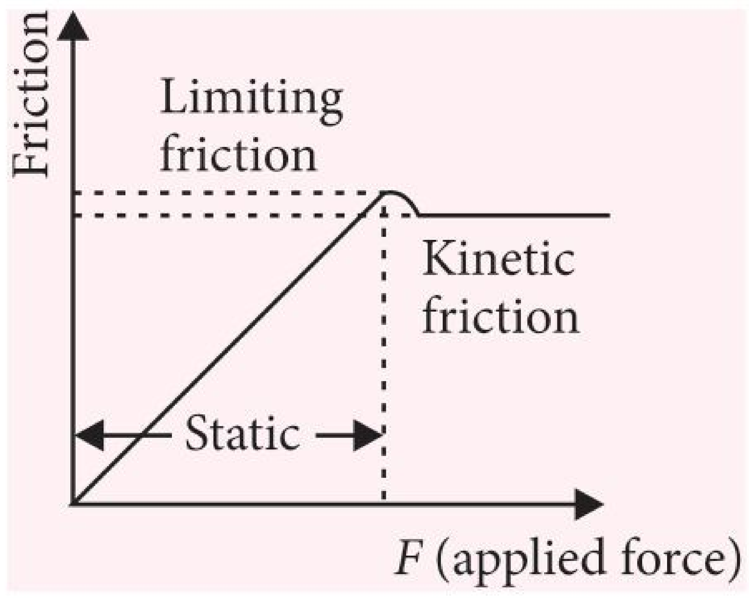


Friction

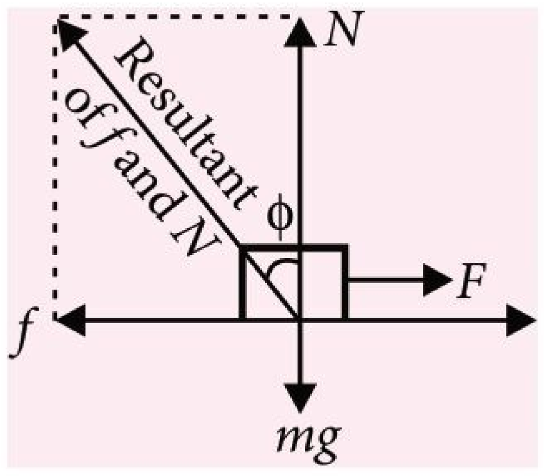
- It is a tangential component of net contact force between two bodies in contact.



- Variation of friction force with applied force

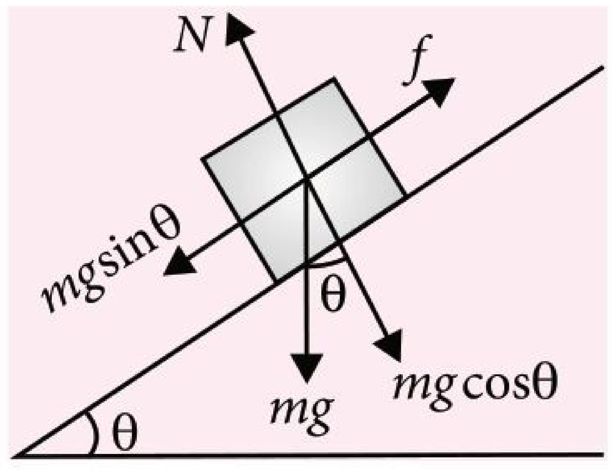


- **Angle of Friction (ϕ) :** The angle which the resultant of the force of limiting friction f and normal reaction N makes with the direction of normal reaction N.



$$\tan\phi = \frac{f}{N} = \mu \text{ or, } \phi = \tan^{-1}\mu$$

- **Angle of Repose :** It is defined as the maximum angle of inclination of a plane with the horizontal at which a body placed on it is at rest.

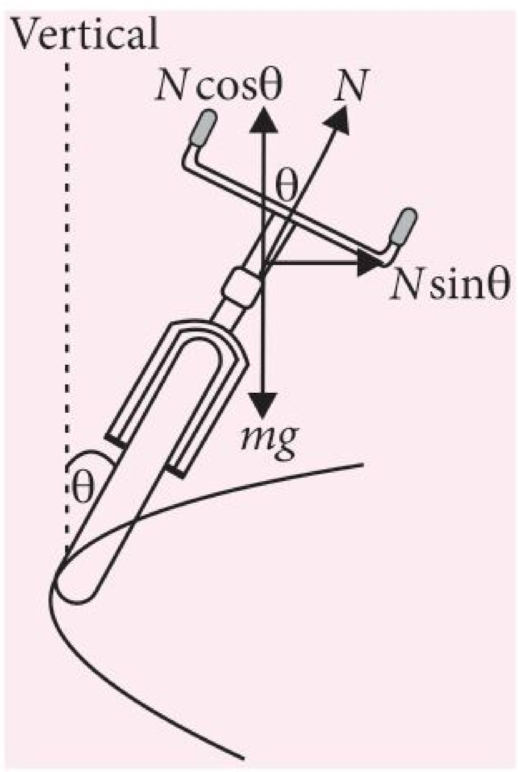
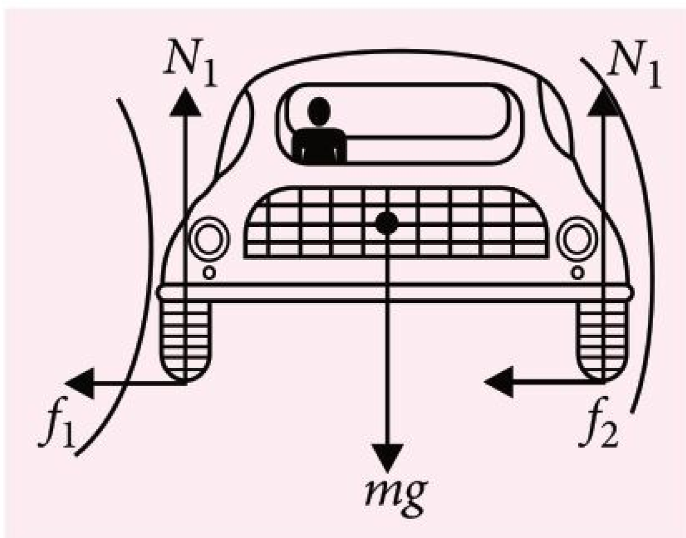
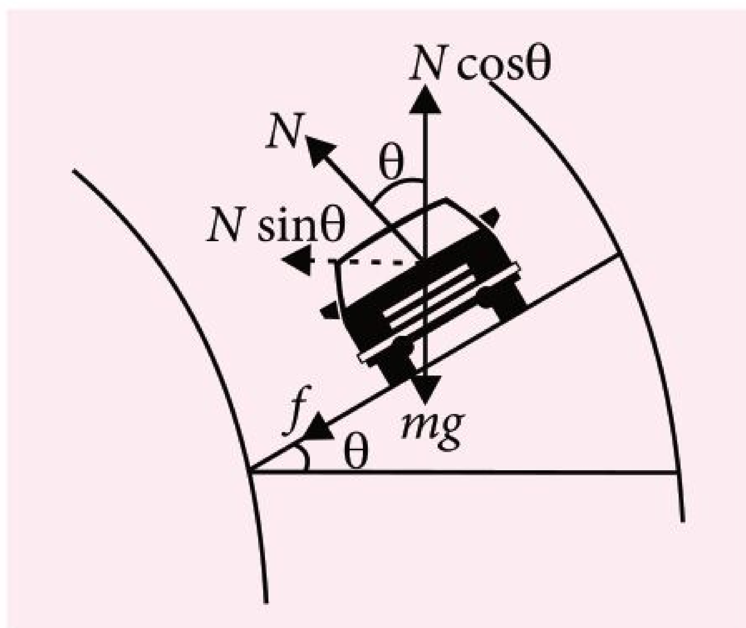


$$f = mg\sin\theta \text{ and } N = mg\cos\theta$$

$$\text{So, } \mu_s = \frac{f}{N} = \frac{mg\sin\theta}{mg\cos\theta} = \tan\theta \Rightarrow \theta = \tan^{-1}\mu_s$$

  - This fact is used for finding the coefficient of static friction in the laboratory.
  - Angle of repose (θ) = Angle of friction (ϕ)

Circular Turnings and Banking of Roads

Bending of cyclist on circular turning for safe going	Motion of a car on a level road	Motion of a car on a banked circular road
 	 	
• Component of normal reaction $N\sin\theta$ provides centripetal force. • Maximum speed of cyclist, $v = \sqrt{rg\tan\theta}$	• Friction force (μN) between tyres and road provides a suitable centripetal force. • Maximum speed of car for safe turning $v = \sqrt{\mu rg}$	• Component of friction force (f cos θ) and component of normal reaction (N sin θ) provide required centripetal force for safe turning. • The maximum permissible speed to avoid slipping $v_{max} = \left[\frac{rg(\mu + \tan\theta)}{1 - \mu\tan\theta}\right]^{1/2}$ • If road is smooth then $\mu = 0$ and $v_{max} = \sqrt{rg\tan\theta}$

Work refers to an activity involving a force and movement in the direction of the force. A force of 20 N pushing an object 5 m in the direction of the force does 100 J of work.

Energy is the capacity for doing work. You must have energy to accomplish work. It is like the currency for performing work. To do 100 J of work, you must expend 100 J of energy.

Power is the rate of doing work or the rate of using energy, which are numerically the same. If you do 100 J of work in 1 s (using 100 J of energy), the power is 100 W.

Work

- Work is said to be done on a body only if a force acts on the body and the point of application of the force moves in the direction of the force.

Work Done

- **By a constant force**

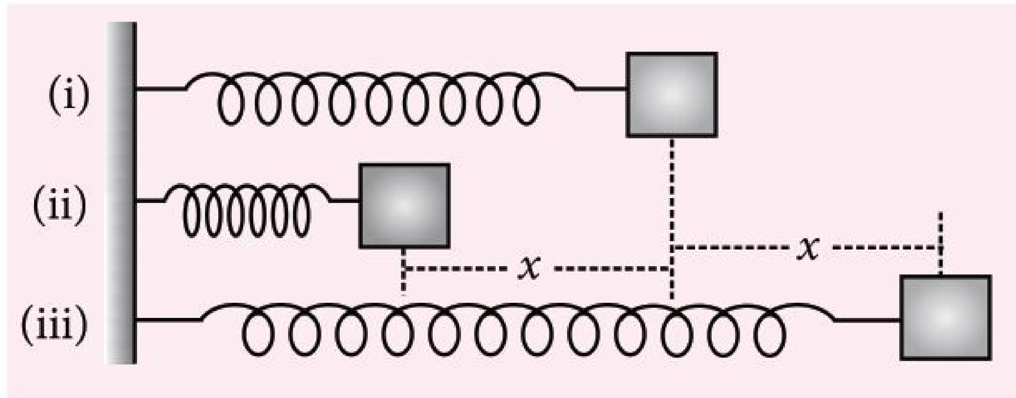
  $W = \vec{F}\cdot\vec{s} = \vec{F}\cdot(\vec{r}_f - \vec{r}_i) = Fs\cos\theta$

  = Force × displacement in the direction of force.
- **By a variable force**

  $W = \int_{x_i}^{x_f} F\cdot dx$, where $F = f(x)$
- **By area under *F*-*x* graph :** If force is a function of x, we can find work done by area under F-x graph with projection along x-axis. Work done can be obtained by area under F-x graph, but sign of work done should be decided by you. If force and displacement both are positive or negative, work done will be positive. If one is positive and other is negative then work done will be negative.

- **Work done by spring-force**
  - $W = \int_{x_1}^{x_2} -kx\,dx = \frac{1}{2}kx_1^2 - \frac{1}{2}kx_2^2$

Initial state of the spring	Final state of the spring	x_1	x_2	W
Natural	Compressed	0	$-x$	$-\frac{1}{2}kx^2$
Natural	Elongated	0	x	$-\frac{1}{2}kx^2$
Elongated	Natural	x	0	$\frac{1}{2}kx^2$
Compressed	Natural	$-x$	0	$\frac{1}{2}kx^2$
Elongated	Compressed	x	$-x$	0
Compressed	Elongated	$-x$	x	0



(i) Spring is in its natural length.
(ii) Spring is elongated by an amount x.
(iii) Spring is compressed by an amount x.

Conservative and Non-conservative Forces

- All central forces are conservative but all conservative forces are not central forces.
- Forces acting along the line joining the centres of two bodies are called central forces. Gravitational and electrostatic forces are central forces.
- Forces acting along the line joining the centres of two bodies are called central forces. Gravitational and electrostatic forces are central forces.
- The concept of potential energy exists only in the case of conservative force.

Conservative force	Non-conservative force
• Work done does not depend on the path.	• Work done depends on the path.
• Work done in a round trip is zero.	• Work done in a round trip is not zero.
• When only a conservative force acts within a system, the kinetic energy and potential energy can change. However their sum, the mechanical energy of the system does not change.	• Work done against a non-conservative force may be dissipated as heat energy. Hence, the mechanical energy of the system changes.
• Work done is completely recoverable.	• Work done is not completely recoverable.

Kinetic Energy

- Kinetic energy (KE) is the capacity of a body to do work by virtue of its motion.

- If a body of mass m has a velocity v its kinetic energy is equivalent to the work which an external force would have to do to bring the body from rest upto its velocity v.

  $KE = \frac{1}{2}mv^2$

  Work done by the constant force = Fs

  $W = (ma)\left(\frac{v^2}{2a}\right) = \frac{1}{2}mv^2$

- Since, both m and v^2 are always positive. KE is always positive and does not depend on the direction of motion of the body.

Potential Energy

- Potential energy is defined only for conservative forces.
- In a conservative force field, difference in potential energy between two points is the negative of work done by conservative forces in displacing the body (or system) from some initial position to final position. Hence,

  $\Delta U = -W$ or $U_B - U_A = -W_{A \to B}$

- Absolute potential energy at a point can be defined with respect to a reference point where potential energy is assumed to be zero. Reference point corresponding to gravitational potential energy and electrostatic potential energy is assumed at infinity. Reference point corresponding to spring potential energy is taken at a point at natural length of spring. Now, negative of work done in displacement of body from reference point (say O) to the point under consideration (say P) is called absolute potential energy at P. Thus,

  $U_P = -W_{O \to P}$

- Potential energy in different conservative fields
  - Gravitation potential energy of a body at height h from earth's surface, $U_g = -W_g = mgh$
  - Electrostatic potential energy between two point charges q_1 and q_2 is

    $U_e = \frac{1}{4\pi\varepsilon_0}\frac{q_1q_2}{r}$;

    r = separation between charge particles.
  - Elastic potential energy : Reference point is taken at natural length of the spring.

    If x is expansion / compression in spring then potential enerrgy of spring (Energy stored in spring) is $U = \frac{1}{2}kx^2$ and spring force $F = -kx$ (directed towards natural length).

Key Point

- Kinetic energy depends on the frame of reference. For example, the kinetic energy of a person of mass m sitting in a train moving with speed v is zero in the frame of train but $\frac{1}{2}mv^2$ in the frame of earth.
- Relation between potential energy and conservative force
- If U is function of only one variable, then

  $F = -\frac{dU}{dr} = -$ slope of U-r graph.

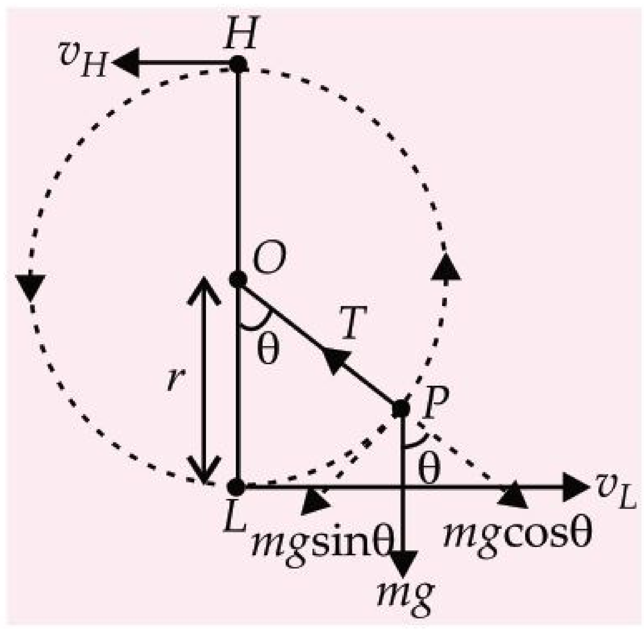
- If U is a function of three coordinate variables x, y and z, then $\vec{F} = -\left[\frac{\partial U}{\partial x}\hat{i} + \frac{\partial U}{\partial y}\hat{j} + \frac{\partial U}{\partial z}\hat{k}\right]$

Work-Energy Theorem

- It states that work done by all forces acting on a body is equal to the change in the kinetic energy of the body.
- This theorem is valid for a system in presence of all types of forces (external or internal, conservative or non-conservative).
- This theorem can be applied to non-inertial frame also.

Motion in Vertical Circle

When a small body of mass m is attached to an inextensible light string of length r and whirling in a vertical circle about a fixed point O to which the other end of the string is attached as shown in figure, then



- Tension at any position of angular displacement, (θ) along a vertical circle is given by $T = \frac{mv^2}{r} + mg\cos\theta$
- Thus, tension at the lowest point ($\theta = 0$) is given by

  $T_L = \frac{mv_L^2}{r} + mg$

  and tension at the highest point ($\theta = 180°$) is given by

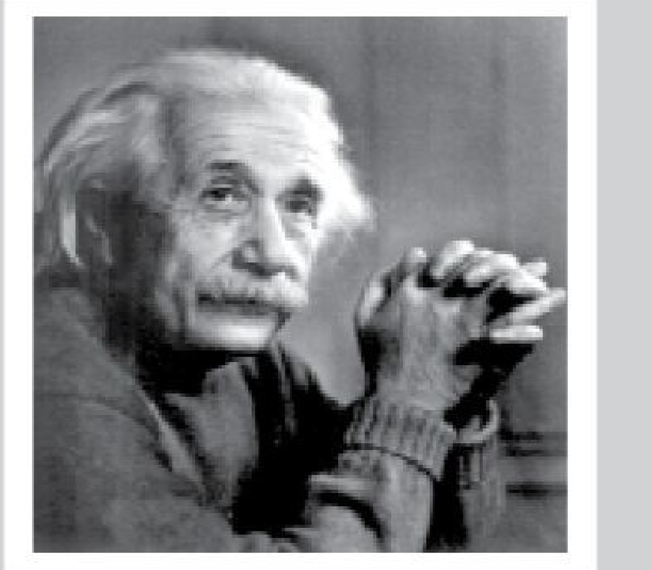
  $T_H = \frac{mv_H^2}{r} - mg$

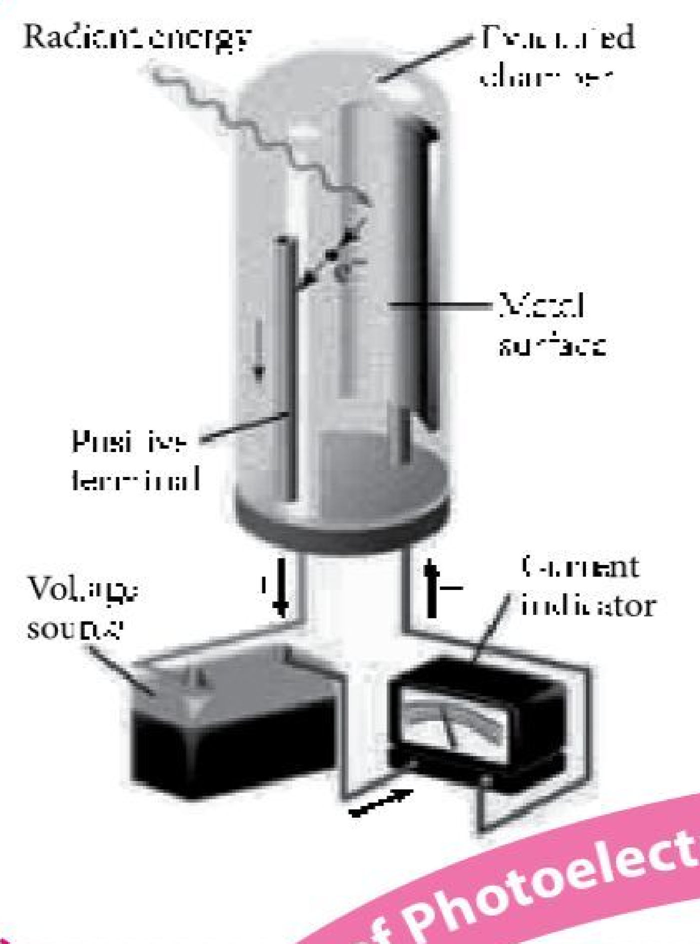
- Minimum velocity at the highest point, $v_H = \sqrt{gr}$
- Minimum velocity at the lowest point for looping the loop, $v_L = \sqrt{5gr}$.
- When the string is horizontal, $\theta = 90°$, minimum velocity, $v = \sqrt{3gr}$.

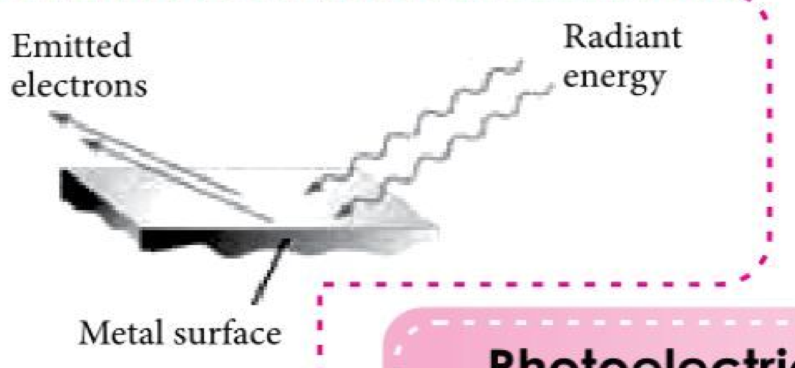
BRAIN MAP

Albert Einstein (1879-1955)

One of the greatest physicists of all time, was born in Ulm, Germany. In 1905, he published three path breaking papers. In the first paper, he introduced the notion of light quanta (now called photons) and used it to explain the features of photoelectric effect. In the second paper, he developed a theory of Brownian motion, confirmed experimentally a few years later and provided a convincing evidence of the atomic picture of matter. The third paper gave birth to the special theory of relativity. In 1921, he was awarded the Nobel Prize in physics for his contribution to theoretical physics and the photoelectric effect.







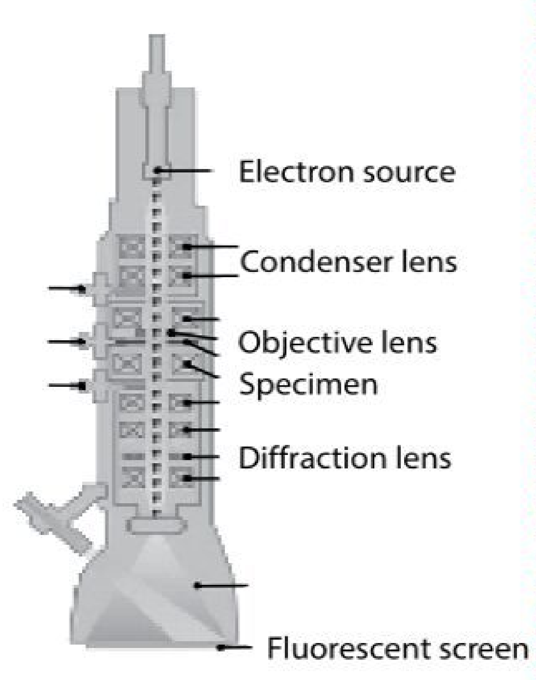
Photoelectric Cell

- An electrical device which converts light energy into electrical energy, is called as photocell or photoelectric cell.
- It works on the principle of photoelectric emission of electrons.

Electron Microscope

- Electron microscope is a device designed to study very minute objects like viruses, microbes and the crystal structure of solids.
- It is based on principle of de Broglie wave and the fast moving electrons can be focussed by E or B field in a same way as beam of light is focussed by glass lenses.

Transmission electron microscope



Application of Photoelectric Effect

Application of de Broglie Waves

DUAL NATURE OF RADIATION AND MATTER

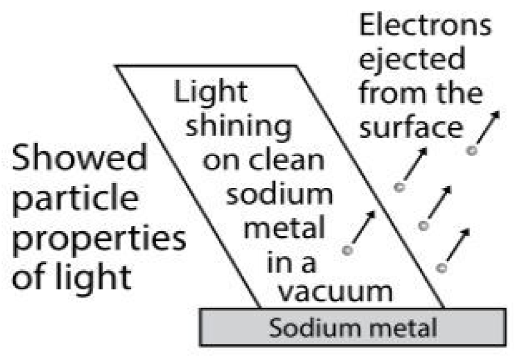
Nature's Love with symmetry arises the matter-wave duality

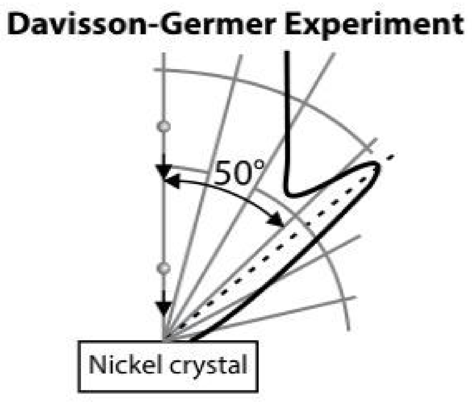
Particle Nature of Radiation

Wave Nature of Matter

Photoelectric Effect

- The phenomena of emission of electrons from a metal surface when an electromagnetic wave of suitable frequency is incident on it is called photoelectric effect.





de Broglie Hypothesis

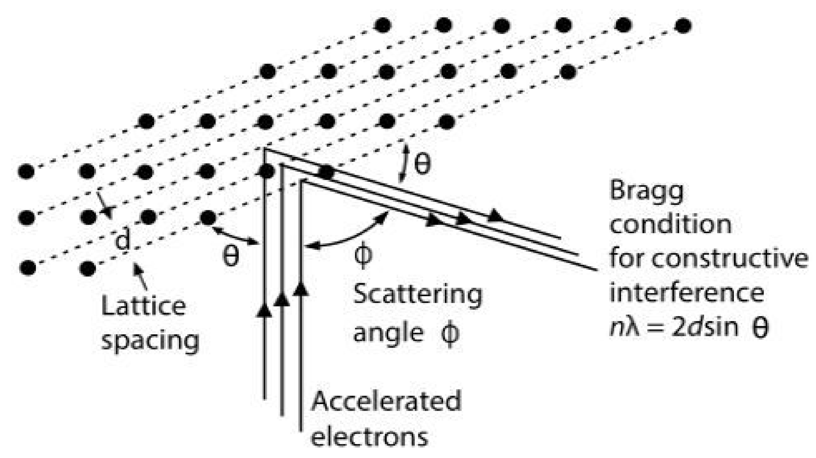
- Due to symmetry in nature, as radiation behaves like a particle, the material particle in motion also possesess wave-like properties. And the waves are called matter waves.

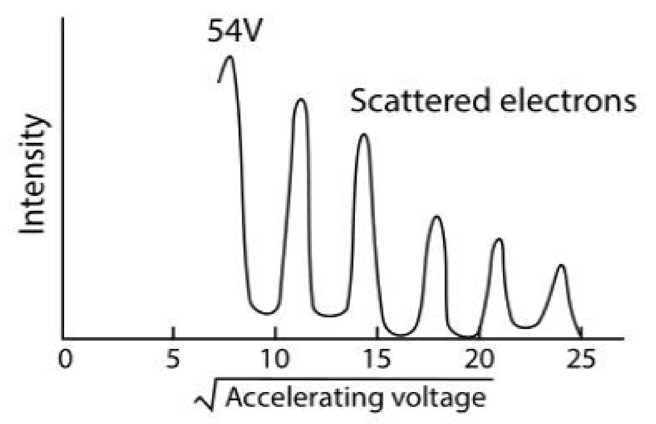
Photoelectric Equation

- $E = K_{max} + \phi_0$; where ϕ_0 = work function, E = energy of incident light, K_{max} = maximum kinetic energy of electrons

$\Rightarrow h\upsilon = \frac{1}{2}mv_{max}^2 + h\upsilon_0$

$\Rightarrow \frac{1}{2}mv_{max}^2 = h(\upsilon - \upsilon_0)$





de Broglie Wavelength

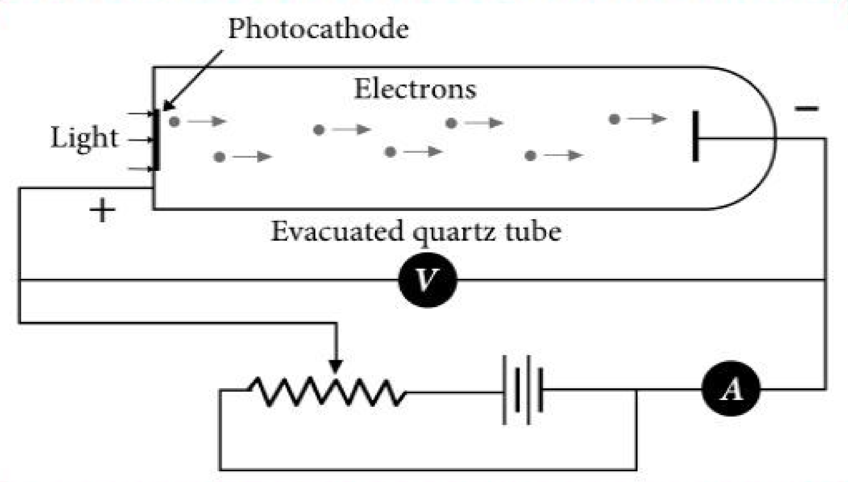
- From quantum theory of radiation, $E = h\upsilon$ and from Einstein-energy of relativistic particles, $E = pc$

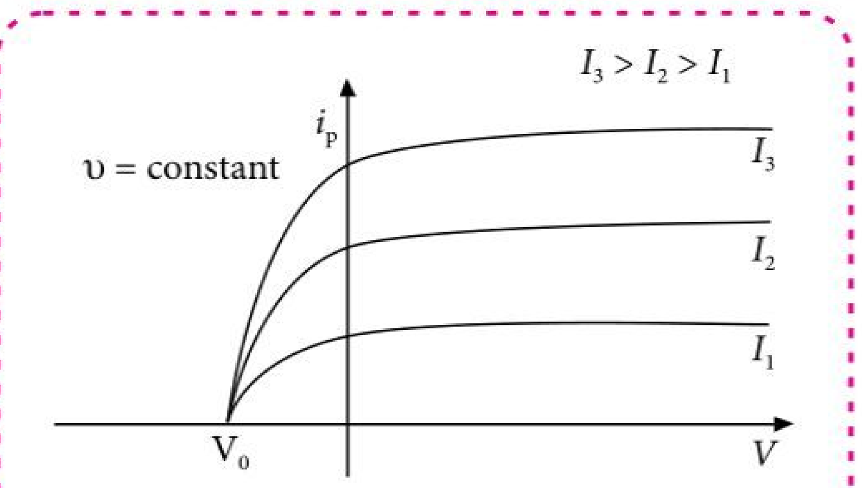
  Combining both, we have de Broglie wavelength, $\lambda = \frac{h}{p} = \frac{h}{mv}$
- de Broglie wavelength in terms of K.E. of electrons,

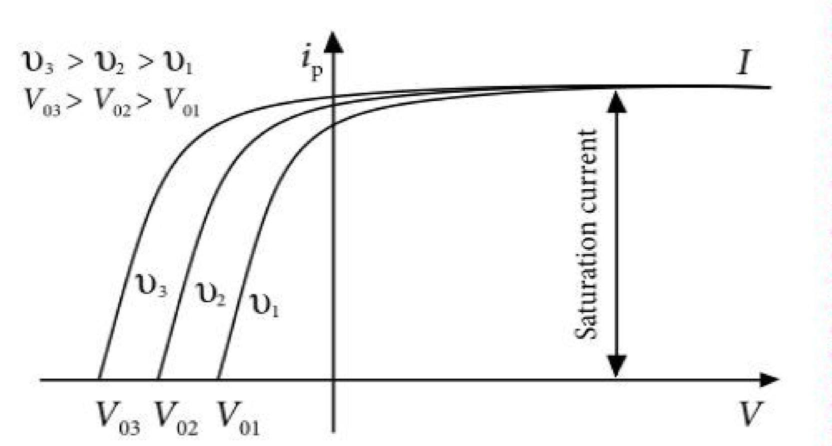
  $\lambda = \frac{h}{\sqrt{2mK}}$, here $p = \sqrt{2mK}$
- de Broglie wavelength of a charged particle having charge q and accelerated by potential V is

  $\lambda = \frac{h}{\sqrt{2qmV}}$ and

  momentum of charged particle, $p = \sqrt{2qmV}$







Experimental Study and Conclusion of Photoelectric Effect

- Photo-current is directly proportional to the intensity of incident light *i.e.*, $i_p \propto I$.

  (At constant frequency υ and potential V)
- At constant frequency and intensity, the minimum negative potential at which the photocurrent becomes zero is called stopping potential (V_0).
- At stopping potential V_0, K_{max} of $e^- = eV_0$
- For a given frequency of the incident radiation, the stopping potential is independent of its intensity.
- The stopping potential varies linearly with the frequency of incident radiation but saturation current value remains constant for fixed intensity.

Davisson Germer Experiment

- **Davisson Germer Experiment:** Study of wave nature of particle.
- At a suitable potential V, the fine beam of electrons from electron gun is allowed to strike on the nickel crystal. The electrons are scattered in all directions and following assumptions were made :
  - ➢ Intensity of scattered electrons depends over scattering angle ϕ.
  - ➢ Always a kink occurs in curve at $\phi = 50°$.
  - ➢ The peak is maximum at accelerating voltage 54 V. After this voltage, peak starts decreasing.
  - ➢ Here, $\theta = \frac{1}{2}(180° - \phi)$

$\Rightarrow \theta = 65°$ at $\phi = 50°$

From Bragg's law (particle nature), $\lambda = 2d \sin\theta \Rightarrow \lambda = 1.65$ Å.

Also, from wave nature at $V = 54$ volt, $\lambda = \frac{12.27}{\sqrt{54}} = 1.65$ Å

- Height through which a body should fall for looping the vertical loop or radius r is, $h = 5r/2$.

☞ Power

Power of a body is defined as the rate at which the body can do the work.

Average power $(P_{av.}) = \dfrac{\Delta W}{\Delta t} = \dfrac{W}{t}$

Instantaneous power $(P_{int.}) = \dfrac{dW}{dt} = \dfrac{\vec{F}\cdot d\vec{s}}{dt}$

[As $dW = \vec{F}\cdot d\vec{s}$]

$P_{inst} = \vec{F}\cdot\vec{v}$

i.e., power is equal to the scalar product of force with velocity.

- The slope of work time curve gives the instantaneous power. As $P = dW/dt = \tan\theta$
- Area under power-time curve gives the work done as $P = \dfrac{dW}{dt}$

$\therefore\ W = \int P\,dt \quad \therefore\ W =$ Area under P-t curve

Collision

- In physics a collision will take place if either of the two bodies come in physical contact with each other or even when path of one body is affected by the force exerted due to the other.

  Collisions are broadly classified into two types :
  - Elastic collision
  - Inelastic collision

Elastic Collision

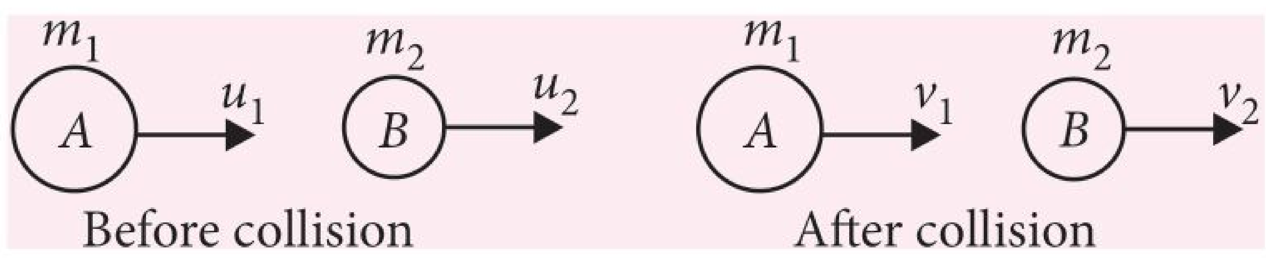
- A collision in which both the momentum and kinetic energy of the body remains conserved. e.g. the collision between two glass balls. The basic characteristics of an elastic collision are :
  - The momentum is conserved.
  - Total energy is conserved.
  - Kinetic energy is conserved.
  - Forces involved in the interaction are of conservative nature.
  - Kinetic energy is not transformed into any other form of energy.

Inelastic Collision

- A collision in which only the momentum of the system is conserved but kinetic energy is not conserved. Most of the collisions in our day to day life are inelastic collisions. *e.g.* mud thrown on the wall. The basic characteristics of an inelastic collision are :
  - Momentum is conserved.
  - Total energy is conserved.
  - Kinetic energy is not conserved.
  - Some or all of the forces involved are non-conservative in nature.
  - A part of the kinetic energy is transformed into other forms of energy.

Elastic Collision in One Dimension

- Consider two bodies A and B of masses m_1 and m_2 moving along the same straight line with velocities u_1 and u_2 respectively. Assume that $u_1 > u_2$ so that two bodies collide. Let v_1 and v_2 be the final velocities of the bodies after collision. The two bodies suffer head on collision and continue moving along the straight line in the same direction as shown in the figure.



According to the law of conservation of momentum

$$m_1u_1 + m_2u_2 = m_1v_1 + m_2v_2$$

Since kinetic energy is conserved in elastic collision, we get

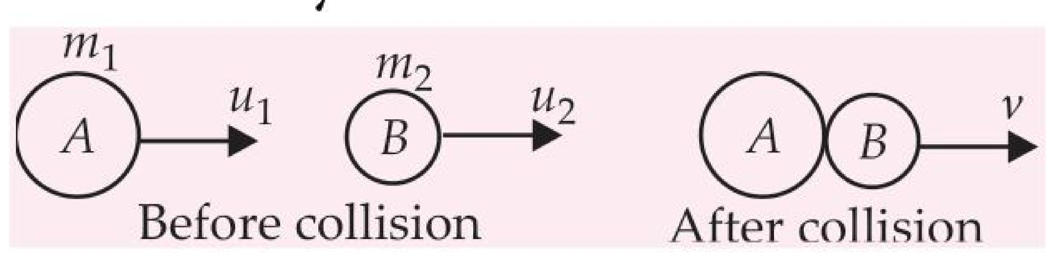
$$\frac{1}{2}m_1u_1^2 + \frac{1}{2}m_2u_2^2 = \frac{1}{2}m_1v_1^2 + \frac{1}{2}m_2v_2^2$$

$$v_1 = \frac{(m_1 - m_2)u_1}{(m_1 + m_2)} + \frac{2m_2u_2}{m_1 + m_2} \quad \text{...(i)}$$

$$v_2 = \frac{2m_1u_1}{m_1 + m_2} + \frac{(m_2 - m_1)}{m_1 + m_2}u_2 \quad \text{...(ii)}$$

Perfectly Inelastic Collision in One Dimension

- Consider two bodies A and B of masses m_1 and m_2 moving with velocities u_1 and u_2 $(u_2 < u_1)$ respectively along the same line collide head on and after collision they have same common velocity v.



According to conservation of linear momentum,

$$m_1u_1 + m_2u_2 = m_1v + m_2v$$

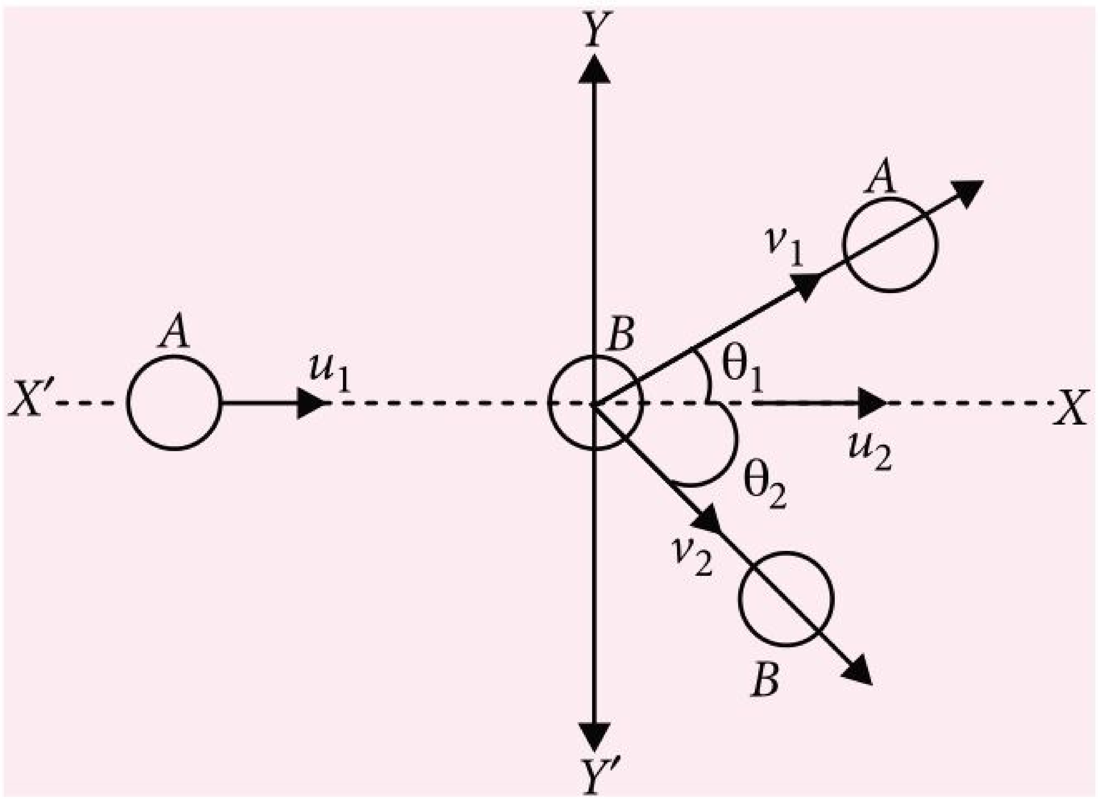
or $$v = \frac{m_1u_1 + m_2u_2}{(m_1 + m_2)} \quad \text{...(i)}$$

Loss in kinetic energy during collision,

$$\Delta K = \frac{1}{2}\frac{m_1m_2}{(m_1 + m_2)}(u_1 - u_2)^2$$

Elastic Collision in Two Dimensions or Oblique Collision

- Let us consider two bodies A and B of masses m_1 and m_2 moving along X-axis with velocities u_1 and u_2 respectively. When $u_1 > u_2$, the two bodies collide. After collision, body A moves with velocity v_1 at an angle θ_1 with X-axis and body B move with a velocity v_2 at an angle θ_2 with X-axis as shown in the figure.



Since the collision is elastic, kinetic energy is conserved.

$$\frac{1}{2}m_1u_1^2 + \frac{1}{2}m_2u_2^2 = \frac{1}{2}m_1v_1^2 + \frac{1}{2}m_2v_2^2 \quad \text{...(i)}$$

Also momentum along X-axis before collision
= momentum after collision along X-axis

$$\Rightarrow m_1u_1 + m_2u_2 = m_1v_1\cos\theta_1 + m_2v_2\cos\theta_2 \quad \text{...(ii)}$$

Similarly along Y-axis

$$0 = m_1v_1\sin\theta_1 - m_2v_2\sin\theta_2 \quad \text{...(iii)}$$

Thus from these three equation (i), (ii) and (iii) we can find the required quantities.

Coefficient of Restitution

- It is defined as the ratio of relative velocity of separation after collision to the relative velocity of approach before collision. It is represented by e.

$$e = \frac{\text{relative velocity of separation (after collision)}}{\text{relative velocity of collision (before collision)}}$$

$$e = \frac{v_2 - v_1}{u_1 - u_2}$$

where u_1, u_2 are velocities of two bodies before collision, and v_1, v_2 are their respective velocities after collision.

☞ Key Point

- When masses of two bodies are equal, *i.e.* $m_1 = m_2 = m$.

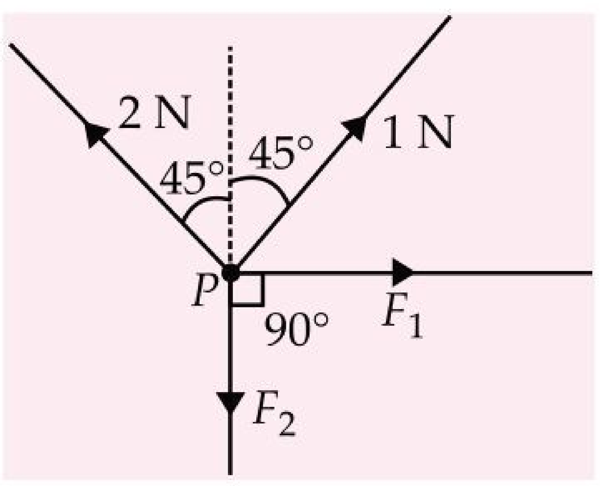
  From equation (i), we get $v_1 = \frac{2mu_2}{2m} = u_2$

  From equation (ii), we get $v_2 = \frac{2mu_1}{2m} = u_1$
- When the body B is initially at rest *i.e.*, $u_2 = 0$.
  From equation (i) and (ii), we get

  $$v_1 = \frac{(m_1 - m_2)u_1}{m_1 + m_2} \quad \text{...(iii)} \qquad v_2 = \frac{2m_1u_1}{m_1 + m_2} \quad \text{...(iv)}$$
- For perfectly elastic collision, $e = 1$.
- For perfectly inelastic collision, $e = 0$.

WRAP it up!

1. There are four forces acting at a point P produced by strings as shown in figure, which is at rest. The forces F_1 and F_2 are



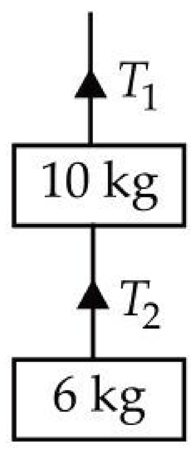
(a) $\frac{1}{\sqrt{2}}$ N, $\frac{3}{\sqrt{2}}$ N

(b) $\frac{3}{\sqrt{2}}$ N, $\frac{1}{\sqrt{2}}$ N

(c) $\frac{1}{\sqrt{2}}$ N, $\frac{1}{\sqrt{2}}$ N

(d) $\frac{3}{\sqrt{2}}$ N, $\frac{3}{\sqrt{2}}$ N

2. A body of mass 6 kg is hanging from another of mass 10 kg as shown in figure. This combination is being pulled up by a string with an acceleration of 2 m s^{-2}. The tension T_1 is (g = 10 m s^{-2})



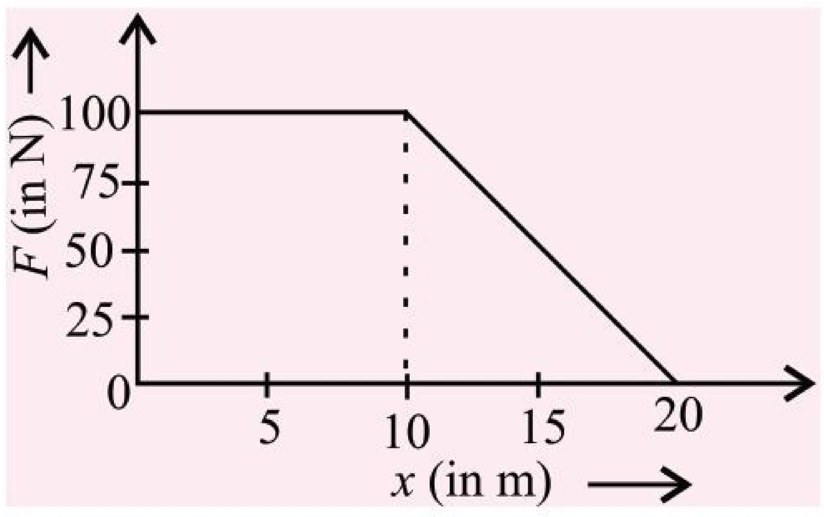
(a) 240 N (b) 150 N
(c) 220 N (d) 192 N

3. A man sits on a chair supported by a rope passing over a frictionless fixed pulley. The man who weighs 1000 N exerts a force of 450 N on the chair downwards, while pulling on the rope. If the chair weighs 250 N and g is 10 m s^{-2}, then the acceleration of the chair is

(a) 0.45 m s^{-2} (b) zero
(c) 9/25 m s^{-2} (d) 2 m s^{-2}

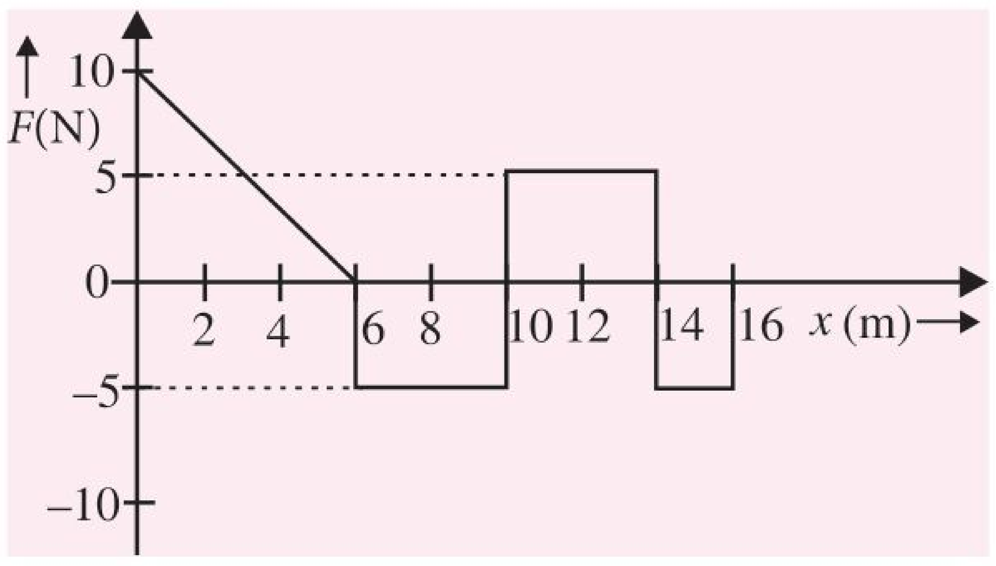
4. A 5 kg shell kept at rest suddenly splits up into three parts. If two parts of mass 2 kg each are found flying due north and east with a velocity of 5 m s^{-1} each, what is the velocity of the third part after explosion?
(a) 10 m s^{-1} due north-east
(b) $10/\sqrt{2}$ m s^{-1} due south-east
(c) $10\sqrt{2}$ ms^{-1} due south-west
(d) $10\sqrt{2}$ ms^{-1} due south-east

5. A force F acting on an object varies with distance x as shown in the figure. The work done by the force in moving the object from $x = 0$ to $x = 20$ m is



(a) 500 J (b) 1000 J (c) 1500 J (d) 2000 J

6. A particle is acted upon by a force F which varies with position x as shown in figure. If the particle at $x = 0$ has kinetic energy of 25 J, then the kinetic energy of the particle at $x = 16$ m is

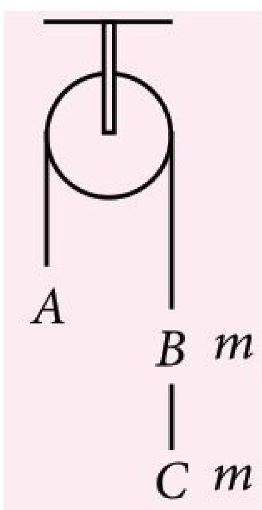


(a) 45 J (b) 30 J (c) 70 J (d) 20 J

7. A stone is dropped from a height h. It hits the ground with a certain momentum P. If the same stone is dropped from a height 100% more than the previous height, the momentum when it hits the ground will change by
(a) 68% (b) 41% (c) 200% (d) 100%

8. A monkey is descending from the branch of a tree with constant acceleration. If the breaking strength of the branch is 75% of the weight of the monkey, then the minimum acceleration with which the monkey can slide down without breaking the branch is
(a) g (b) $g/2$ (c) $g/4$ (d) $3g/4$

9. Three blocks A, B and C each of mass m are attached to a string, passing over a smooth pulley. What is the tension in the string connecting A and B?



(a) $2/3mg$ (b) mg
(c) $4/3mg$ (d) $5/3mg$

10. A block of mass 'm' is kept on a smooth inclined plane of inclination θ. The whole system is given a horizontal acceleration 'a' so that the block remains stationary on the inclined plane. What is the force exerted by the inclined plane on the block?
(a) mg (b) $mg\cos\theta$
(c) $mg\sin\theta$ (d) $\frac{mg}{\cos\theta}$

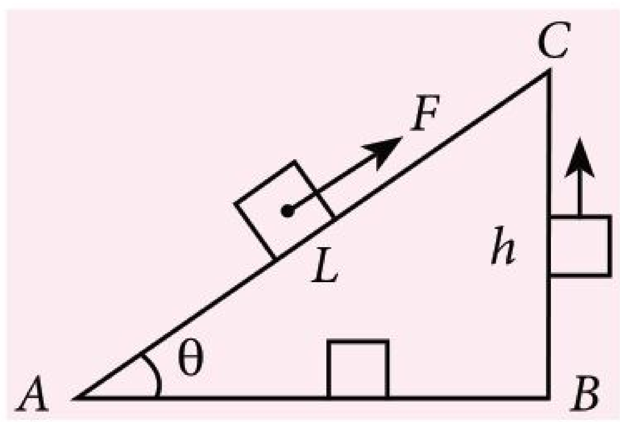
11. A sphere of mass m moving with a constant velocity u hits another stationary sphere of the same mass and of coefficient of restitution (e). The ratio of velocities of the two spheres, after collision, will be
(a) $\frac{e}{(e+1)}$ (b) $\frac{(1-e)}{(1+e)}$ (c) $\frac{1}{e}$ (d) $\frac{(e+1)}{e}$

12. A ball moving with a velocity of 6 ms^{-1} strikes an identical stationary ball. After collision each ball moves at angle of 30° with the original line of motion. Assuming that the collision is elastic, what are the speeds of the balls after the collision?
(a) $\frac{\sqrt{3}}{2}$ m s^{-1}, $2\sqrt{3}$ m s^{-1}
(b) 3 m s^{-1}, $2\sqrt{3}$ m s^{-1}
(c) $2\sqrt{3}$ m s^{-1}, $2\sqrt{3}$ m s^{-1}
(d) $\sqrt{3}$ m s^{-1}, $\sqrt{3}$ m s^{-1}

13. A block of mass 10 kg is moving in x-direction with a constant speed of 10 m s^{-1}. It is subjected to a retarding force $F_r = -0.1x$ J m^{-1} during its travel from $x = 20$ m to $x = 30$ m. Its final kinetic energy will be
(a) 250 J (b) 275 J (c) 450 J (d) 475 J

14. A box of mass 50 kg is pulled up on an incline 12 m long and 2 m high by a constant force of 100 N from rest. it acquires a velocity of 2 m s^{-1} on reaching the top. Work done against friction is
(Take $g = 10$ m s^{-2})
(a) 50 J (b) 100 J (c) 150 J (d) 200 J

15. If W_{AC} and W_{ABC} denote the work done by gravity in taking a body on mass m from A to C in the gravitational field via the paths AC and $A \to B \to C$, along a smooth inclined plane, then



(a) $W_{AC} < W_{ABC}$
(b) $W_{AC} = W_{ABC}$
(c) $W_{AC} < W_{ABC}$
(d) $W_{AC} = (\sqrt{L^2 - h^2})W_{ABC}$

16. In an NCC training camp, a cadet fired a bullet of mass 50 gram with a speed of 200 m s^{-1}, on a soft plywood board of thickness 20 mm. It was found that the kinetic energy of the emerging bullet was only 25% of its initial K.E. What is the percentage decrease in the speed of the bullet?
(a) 25% (b) 50% (c) 60% (d) 75%

17. A particle of mass m moving with a speed v towards east strikes another particle of the same mass moving with same speed v towards north. After striking the two particles stick together and move in north east direction. With what speed this new particle of mass 2 m will move in north-east direction?
(a) v (b) $v/2$ (c) $v/\sqrt{2}$ (d) $v\sqrt{2}$

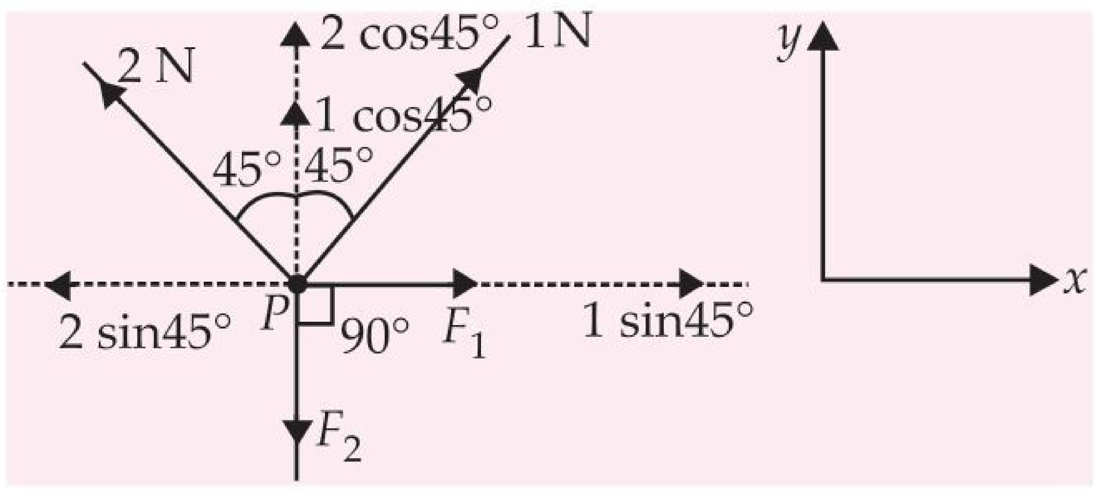
18. A man of mass 50 kg is standing in a gravity free space at a height of 10 m above the floor. He throws a stone of 0.5 kg mass downwards with a speed 2 m s^{-1}. What is the distance of the man above the floor when the stone reaches the floor?
(a) 10 m (b) 10.1 m (c) 9.9 m (d) 20 m

19. Particle A makes a perfectly elastic collision with another particle B at rest. They fly apart in opposite directions with equal speeds. What is the ratio of their masses m_A/m_B?
(a) 1/2 (b) 1/3 (c) 1/4 (d) $1/\sqrt{3}$

20. Water is flowing in a river at 2.0 m s^{-1}. The river is 50 m wide and has an average depth of 5.0. What is the power available from the flow of water in the river? (Density of water = 1000 kg m^{-3})
(a) 0.5 MW (b) 1.0 MW
(c) 1.5 MW (d) 2.0 MW

SOLUTIONS

1. (a):



Applying equilibrium conditions, $\Sigma F_x = 0$

$\Rightarrow$ $F_1 + 1\sin45° - 2\sin45° = 0$

or $F_1 = 2\sin45° - 1\sin45°$

$$= \frac{2}{\sqrt{2}} - \frac{1}{\sqrt{2}} = \frac{2-1}{\sqrt{2}} = \frac{1}{\sqrt{2}} \text{ N}$$

and $\Sigma F_y = 0 \Rightarrow 1\cos45° + 2\sin45° - F_2 = 0$

$$F_2 = \frac{2}{\sqrt{2}} + \frac{1}{\sqrt{2}} = \frac{2+1}{\sqrt{2}} = \frac{3}{\sqrt{2}} \text{ N}$$

2. (d)

3. (d): Let a be the acceleration of the man and the chair and T be the tension in the rope.

For upward motion of the man,

$$T + 450 - 1000 = \left(\frac{1000}{10}\right)a$$

or $T = 550 + 100a$ ...(i)

For upward motion of the chair,

$$T - 450 - 250 = \left(\frac{250}{10}\right)a$$

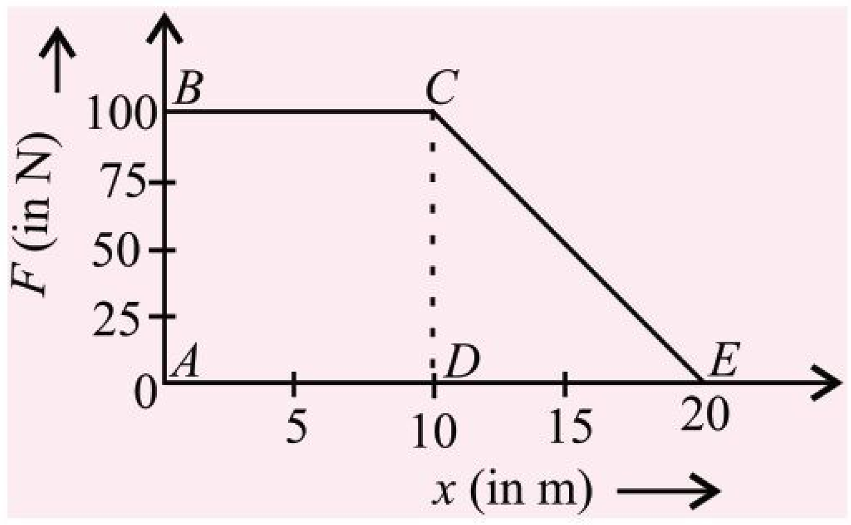
or $T = 25a + 700$ ...(ii)

Equating (i) and (ii), we get

$550 + 100a = 25a + 700$; a = 2 m s^{-2}

4. (c)

5. (c):

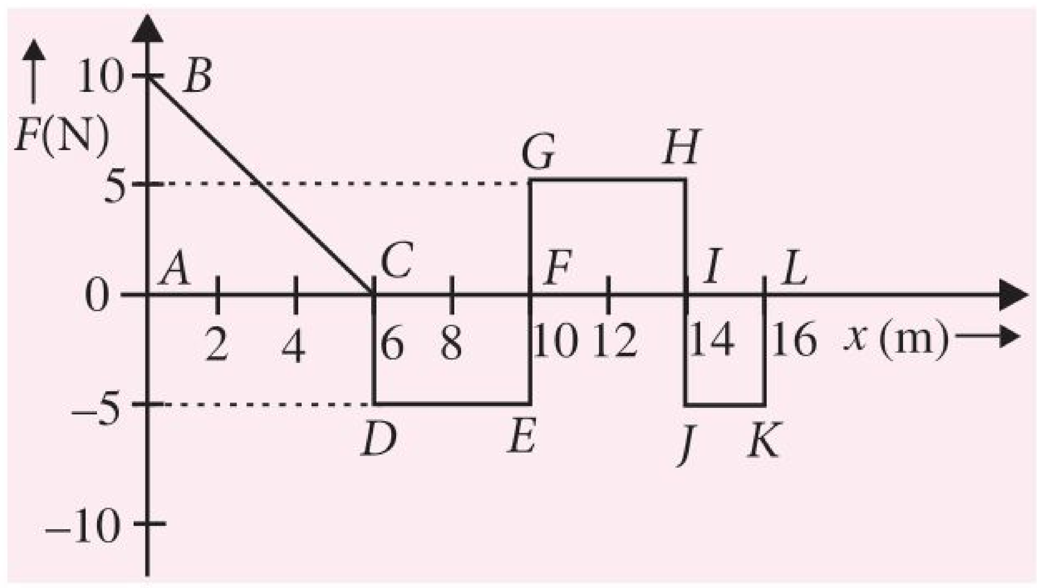


Work done = Area under F-x graph

W = Area of rectangle $ABCD$ + Area of ΔCDE

$$= 100\times10+\frac{1}{2}\times10\times100 = 1000 + 500 = 1500 \text{ J}$$

6. (a):



Work done W = Area under F-x graph with proper sign

W = Area of triangle ABC + Area of rectangle $CDEF$ + Area of rectangle $FGHI$ + Area of rectangle $IJKL$

$$W = \left[\frac{1}{2}\times6\times10\right]+\left[4\times(-5)\right]+\left[4\times(5)\right]+\left[2\times(-5)\right]$$

= 30 – 20 + 20 – 10 = 20 J … (i)

According to work energy theorem

$K_f - K_i = W$

or $(K_f)_{x=16\text{ m}} - (K_i)_{x=0\text{ m}} = W$

$(K_f)_{x=16\text{ m}} = (K_i)_{x=0\text{ m}} + W = 25\text{ J} + 20\text{ J} = 45\text{ J}$

7. (b): When a stone is dropped from a height h hits the ground, it velocity $v_1 = \sqrt{2gh}$ $\because v^2 = u^2 + 2gh$

$\therefore$ Its momentum $P_1 = mv_1 = m\sqrt{2gh}$ ...(i)

and then it is dropped from a height $2h$ (100% more than the previous height) and when it hits the ground, its velocity $v_2 = \sqrt{2g\cdot 2h}$

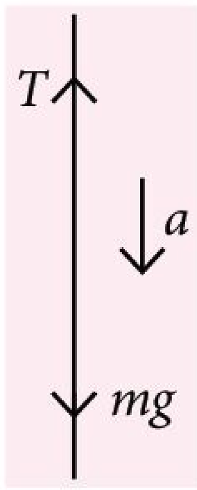
and its momentum $P_2 = mv_2 = m\sqrt{2\cdot(2gh)} = \sqrt{2}P_1$

$\therefore$ Percentage change in momentum

$$= \left(\frac{P_2 - P_1}{P_1}\right)\times 100\% = \left(\frac{\sqrt{2}P_1 - P_1}{P_1}\right)\times 100\% = 41\%$$

8. (c) : The breaking strength of the branch of the tree

$= 75\%$ (weight of the monkey) $= \frac{3}{4}mg$



When the monkey descends from the branch of the tree, with an acceleration (a)

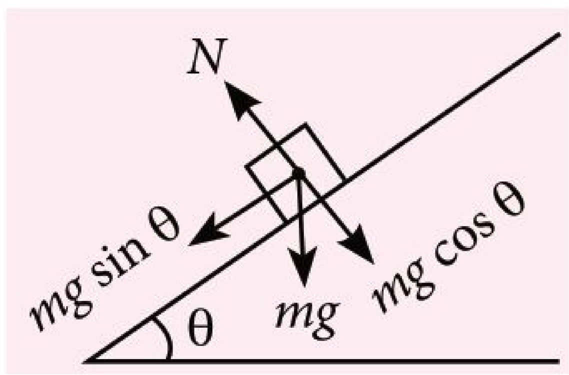
$mg - T = ma$

$\therefore$ $mg - \frac{3}{4}mg = m \times a$; $a = g/4$

9. (a)

10. (b): Here, the force exerted by the plane on the block is the normal force N of the block So, from free body diagram,

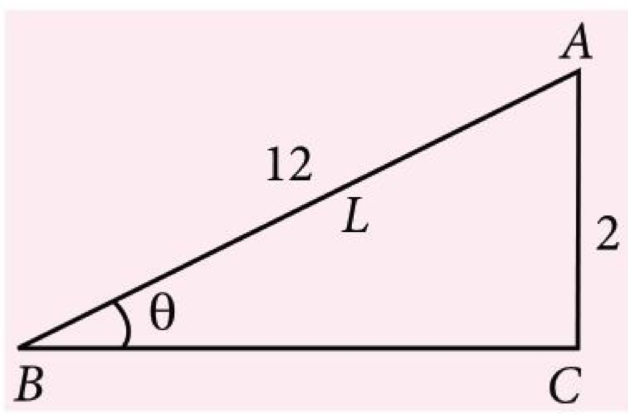
$N = mg\cos\theta$



11. (b) **12. (c)** **13. (d)**

14. (b): For the inclined plane, $\sin\theta = \frac{2}{12} = \frac{1}{6}$

When the box reaches its highest point A, its total energy



$= \text{P.E.} + \text{K.E.} = \frac{1}{2}mv^2$

$= 50\times 10\times 2 + \frac{1}{2}\times 50\times 2\times 2$

$= 1000 + 100 = 1100$ J ...(i)

The upward motion of the block is opposed by the force of friction.

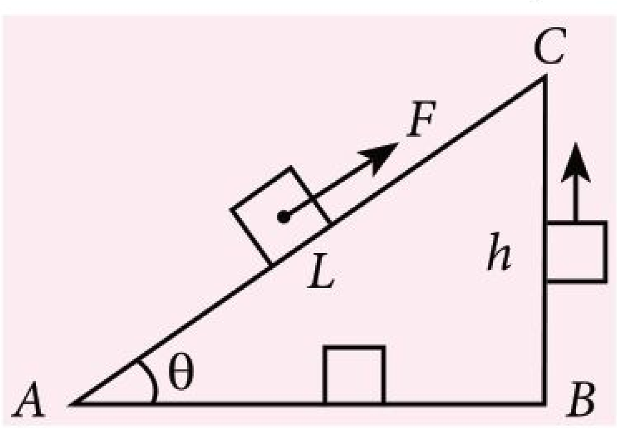
Resultant work = Work down by the applied force (W_1) – Work done against friction (W_2)

$= F \times L - W_2 = 100 \times 12 - W_2$ ...(ii)

$\therefore$ From (i) and (ii), $1100\text{ J} = 1200 - W_2$

$\therefore$ $W_2 = 1200 - 1100 = 100$ J

15. (b):



The work done in taking the body form A to C along the path AC

$W_{AC} = F \times L = mg\sin\theta \times L = mgh$ $\left(\because \sin\theta = \frac{h}{L}\right)$

For the path AB, $W_{AB} = F\cos 90° \times AB = 0$

For the path BC, $W_{BC} = mgh$

$\therefore$ $W_{AB} + W_{BC} = W_{AC}$

i.e. in a conservative field, the work done does not depend upon the path. It depends only upon the initial and final positions.

16. (b) : The initial K.E. of the bullet

$= \frac{1}{2}mv^2 = \frac{1}{2}\times 50\times 10^{-3}\times 200\times 200$

$\therefore$ $K_i = 1000$ J

$\therefore$ Final K.E. $(K_f) = \frac{25}{100} \times 1000 = 250$ J

$\Rightarrow$ $\frac{1}{2}mv_f^2 = 250 \Rightarrow v_f = 100\text{ m s}^{-1}$

$\therefore$ Decrease in speed = (200 – 100) = 100 m s^{-1}

$\therefore$ Percentage decrease in speed $= \frac{100}{200}\times 100 = 50\%$

17. (c)

18. (b): The time taken by the stone to reach the floor is $t = \frac{s}{u} = \frac{10}{2} = 5\text{ s}$

The acceleration due to gravity is not to be considered in the gravity free space.

Similar to bullet-rifle problem, we will apply the conservation of linear momentum.

Momentum of the stone + momentum of the man = 0

or $m_1u_1 + m_2u_2 = 0$

$\therefore$ $0.5 \times 2 = -50 \times u_2$; $u_2 = -\frac{1}{50}\text{ m s}^{-1}$

$\therefore$ The -ve sign shown the upward velocity of the man.

$\therefore$ In 5 s, the man will move upwards through a distance of $\frac{1}{50}\times 5 = 0.1$ m

Thus, distance of man above the floor $h = 10 + 0.1 = 10.1$ m

19. (b)

20. (b): $\text{Power} = \frac{\text{Work}}{\text{Time}} = \frac{\text{K.E.}}{\text{Time}}$

Thus, Power = K.E. of the water flowing per second

$= \frac{1}{2}\frac{mv^2}{t}$

But velocity = Distance travelled/s = 2 m s^{-1}

and $\frac{\text{mass}}{t} = \frac{\text{volume}}{t}\times\text{density} = L\times b\times h\times d$

$\therefore$ $\frac{m}{t} = 2 \times 50 \times 5 \times 10^3 = 5 \times 10^8\text{ kg s}^{-1}$

[L = 2 m = distance travelled in one second]

$\therefore$ $P = \frac{1}{2}\left(\frac{mv^2}{t}\right) = \frac{1}{2}\times 5\times 10^5\times 2\times 2 = 1\text{ MW}$

◈◈

CBSE warm-up!

CLASS-XI

TERM-I OBJECTIVE TYPE QUESTIONS*

Series 3 — Gravitation

GENERAL INSTRUCTIONS

1. The Question Paper contains three sections.
2. Section A has 25 questions. Attempt any 20 questions.
3. Section B has 24 questions. Attempt any 20 questions.
4. Section C has 6 questions. Attempt any 5 questions.
5. All questions carry equal marks.
6. There is no negative marking.

Time allowed : 90 minutes — ***Maximum marks : 35***

SECTION-A

This section consists of 25 multiple choice questions with overall choice to attempt any 20 questions. In case more than desirable number of questions are attempted, ONLY first 20 will be considered for evaluation.

1. A satellite is launched into a circular orbit of radius R around the earth. A second satellite is launched into an orbit of radius 1.01 R. The period of the second satellite is longer than that of the first by approximately

(a) 1.5 % (b) 3.0%
(c) 1.0% (d) 0.5%

2. A particle of mass 10 g is kept on the surface of a uniform sphere of mass 100 kg and radius 10 cm. Find the work to be done against the gravitational force between them to take the particle far away from the sphere. You may
(Take $G = 6.67 \times 10^{-11}$ N m^2 kg^{-2})

(a) 13.34×10^{-10} J (b) 3.33×10^{-10} J
(c) 6.67×10^{-9} J (d) 6.67×10^{-10} J

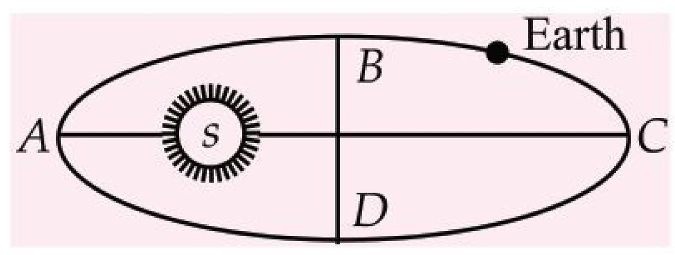
3. The escape velocity from a spherical satellite is V_e. The escape velocity from another satellite of double the radius and half the mean density will be

(a) $\frac{V_e}{2}$ (b) $2V_e$
(c) $\sqrt{2}V_e$ (d) $\frac{V_e}{3}$

4. In planetary motion, the angular momentum conservation leads to the law of

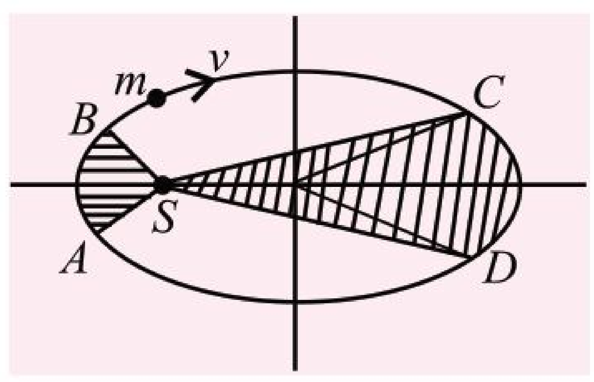
(a) orbits
(b) areas
(c) periods
(d) conservation of kinetic energy

5. The earth rotates around the sun (see figure) in an elliptical orbit. At which point will velocity be maximum?



(a) At A (b) At B
(c) At C (d) At D

6. The figure shows elliptical orbit of a planet m about the sun S. The shaded area SCD is twice the shaded area SAB. If t_1 is the time



*Chapterwise practice questions for CBSE Exam Term- I as per the pattern issued by CBSE.

for the planet to move from C to D and t_2 is the time to move from A to B then

(a) $t_1 = 4t_2$ (b) $t_1 = 2t_2$
(c) $t_1 = t_2$ (d) $t_1 > t_2$

7. Which of the following statements is correct regarding a geostationary satellite?

(a) A geostationary satellite goes around the earth in east-west direction.
(b) A geostationary satellite goes around the earth in west-east direction.
(c) The time-period of a geostationary satellite is 48 hours.
(d) The angle between the equatorial plane and the orbital plane of geostationary satellite is 90°.

8. If both mass and the radius of the earth decrease by 1%, then the value of the acceleration due to gravity will

(a) decrease by 1% (b) increase by 1%
(c) increase by 2% (d) remain unchanged.

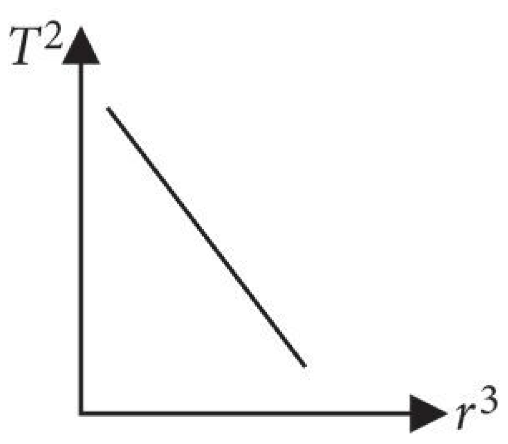
9. A high jumper can jump 2.0 m on the earth. With the same effort how high will he be able to jump on a planet whose density is one-third and radius one-fourth those of the earth?

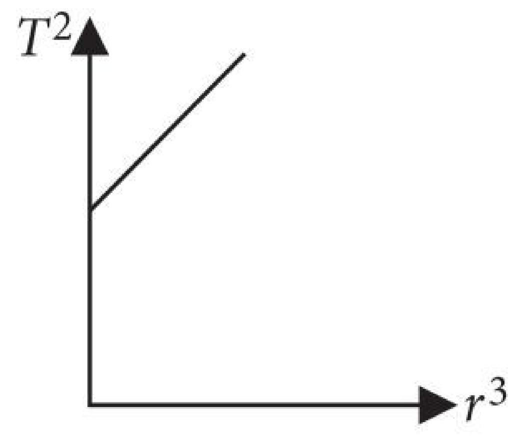
(a) 4 m (b) 8 m
(c) 12 m (d) 24 m

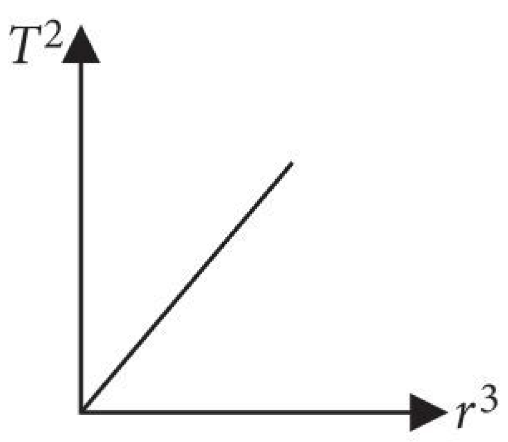
10. Let A be the area swept out by the line joining the earth and the sun during February 1991. The area swept out by the line during a typical week in February 1991 is

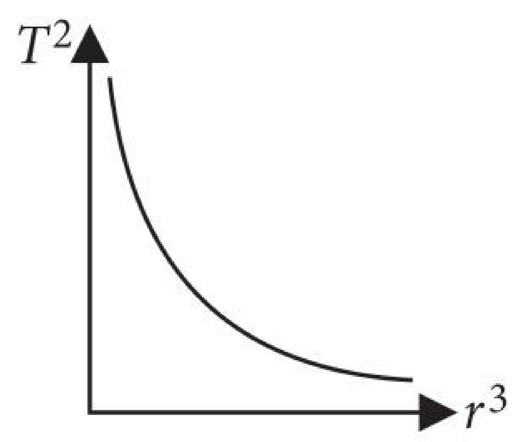
(a) A (b) $2A$ (c) $4A$ (d) $\frac{A}{4}$

11. Which of the following graphs between the square of the time period and cube of the distance of the planet from the sun is correct?

(a)


(b)


(c)


(d)


12. A comet of mass m moves in a highly elliptical orbit around the sun of mass M. The maximum and minimum distances of the comet from the centre of the sun are r_1 and r_2, respectively. The magnitude of angular momentum of the comet with respect to the centre of sun is

(a) $\left[\frac{GMr_1}{(r_1+r_2)}\right]^{1/2}$ (b) $\left[\frac{GMmr_1}{(r_1+r_2)}\right]^{1/2}$

(c) $\left[\frac{2Gm^2r_1r_2}{(r_1+r_2)}\right]^{1/2}$ (d) $\left[\frac{2GMm^2r_1r_2}{(r_1+r_2)}\right]^{1/2}$

13. The period of revolution of an earth's satellite close to surface of earth is 90 min. The time period of another satellite in an orbit at a distance of four times the radius of earth from its surface will be

(a) $90\sqrt{9}$ min (b) 270 min
(c) 720 min (d) 360 min

14. A satellite in a circular orbit of radius R has a period of 4 h. Another satellite with orbital radius $3R$ around the same planet will have a period (in h)

(a) 16 (b) 4 (c) $4\sqrt{27}$ (d) $4\sqrt{8}$

15. The universal law of gravitation is the force law also known as the

(a) triangular law (b) square law
(c) inverse square law (d) parallelogram law

16. Two identical spheres of radius R made of the same material are kept at a distance d apart. Then the gravitational attraction between them is proportional to

(a) d^{-2} (b) d^2 (c) d^4 (d) d

17. The magnitudes of the gravitational field at distances r_1 and r_2 from the centre of a uniform sphere of radius R and mass M are F_1 and F_2 respectively. Then

(a) $\frac{F_1}{F_2} = \frac{r_1}{r_2}$, if $r_1 < R$ and $r_2 < R$

(b) $\frac{F_1}{F_2} = \frac{r_2}{r_1}$, if $r_1 < R$ and $r_2 < R$

(c) $\frac{F_1}{F_2} = \frac{r_1^3}{r_2^3}$, if $r_1 > R$ and $r_2 > R$

(d) $\frac{F_1}{F_2} = \frac{r_1^2}{r_2^2}$, if $r_1 > R$ and $r_2 > R$

18. The acceleration due to gravity decreases by Δg_1 when a body is taken to a small height $h << R$. The acceleration due to gravity decreases by Δg_2 when the body is taken to a depth h from the surface of the earth, then (R = Radius of the earth)

(a) $\Delta g_1 = \Delta g_2$ (b) $\Delta g_1 = 2\Delta g_2$
(c) $\Delta g_2 = 2\Delta g_1$ (d) $\Delta g_1 = 4\Delta g_2$

19. Assuming that the earth is a sphere of uniform mass density, what is the percentage decrease in the weight of a body when taken to the end of the tunnel 32 km below the surface of the earth?
(Radius of earth = 6400 km)

(a) 0.25% (b) 0.5% (c) 0.7% (d) 1%

20. The height of the point vertically above the earth's surface at which the acceleration due to gravity becomes 1% of its value at the surface is (R is the radius of the earth)

(a) $8R$ (b) $9R$ (c) $10R$ (d) $20R$

21. A uniform solid sphere of radius R produces a gravitational acceleration a_g on its surface. At what two distances from the centre of the sphere the acceleration due to gravity is $a_g/4$?

(a) $4R$, $0.50R$ (b) $2R$, $0.25R$
(c) $3R$, $0.33R$ (d) $2R$, $0.50R$

22. At what height h above the earth's surface, the value of g becomes $g/2$?
(where R is the radius of the earth)

(a) $(\sqrt{2}-1)R$ (b) $\sqrt{2}R$
(c) $(\sqrt{2}+1)R$ (d) $R\sqrt{2}$

23. Let g_p and g_E be the accelerations due to gravity at the poles and the equator respectively. Assuming that the earth is a sphere of radius R, rotating about its axis with angular velocity ω, then $g_p - g_E$ is equal to

(a) $\frac{\omega^2}{R}$ (b) $R\omega^2$ (c) $R^2\omega^2$ (d) $\frac{\omega^2}{R^2}$

24. The earth is assumed to be a sphere of radius (R) rotating about its axis with an angular speed ω. If the effective acceleration due to gravity at a certain latitude (λ) is $g_p - \frac{3}{4}\omega^2 R$, where g_p is the acceleration due to gravity at the poles, then the value of λ is

(a) 45° (b) 30° (c) 90° (d) 90°

25. If distance between earth and sun become four times, then time period becomes

(a) 4 times (b) 8 times
(c) 1/4 times (d) 1/8 times

SECTION-B

This section consists of 24 multiple choice questions with overall choice to attempt any 20 questions. In case more than desirable number of questions are attempted, ONLY first 20 will be considered for evaluation.

26. The depth at which the value of acceleration due to gravity becomes $\frac{1}{n}$ times the value at the surface is (R be the radius of the earth).

(a) $\frac{R}{n}$ (b) $\frac{R}{n^2}$
(c) $\frac{R(n-1)}{n}$ (d) $\frac{Rn}{(n-1)}$

27. The radii of two planets are respectively R_1 and R_2 and their densities are respectively ρ_1 and ρ_2. The ratio of the accelerations due to gravity (g_1/g_2) at their surfaces is

(a) $\frac{R_1\rho_2}{R_2\rho_1}$ (b) $\frac{R_1\rho_1}{R_2\rho_2}$ (c) $\frac{\rho_1 R_2^2}{\rho_2 R_1^2}$ (d) $\frac{R_1R_2}{\rho_1\rho_2}$

28. A body weighs 72 N on the surface of earth. What is the gravitational force on it due to earth at a height equal to half the radius of the earth from the surface?

(a) 72 N (b) 28 N (c) 16 N (d) 32 N

29. The height at which the acceleration due to gravity decreases by 36% of its value on the surface of the earth is (The radius of the earth is R.)

(a) $\frac{R}{6}$ (b) $\frac{R}{4}$ (c) $\frac{R}{2}$ (d) $\frac{2}{3}R$

30. Which of the following statements is correct regarding the universal gravitational constant G?

(a) G has same value in all systems of units.
(b) The value of G is same everywhere in the universe.
(c) The value of G was first experimentally determined by Johannes Kepler.
(d) G is a vector quantity.

31. Both earth and moon are subjected to the gravitational force of the sun. As observed from the sun, the orbit of the moon

(a) will be elliptical.
(b) will not be strictly elliptical because the total gravitational force on it is not central.
(c) is not elliptical but will necessarily be a closed curve.
(d) deviates considerably from being elliptical due to influence of planets other than earth.

32. The mass of the earth is 6.0×10^{24} kg. The potential energy of a body of mass 50 kg at a distance of 6.3×10^9 m from the centre of the earth is
(a) -3.23×10^9 J (b) -3.19×10^6 J
(c) -2.5×10^6 J (d) -4.0×10^{11} J

33. Two spheres of masses m and M are situated in air and the gravitational force between them is F. The space around the masses is now filled with a liquid of specific gravity 3. The gravitational force will now be
(a) $3F$ (b) F (c) $F/3$ (d) $F/9$

34. Work done in taking a mass from one point to another in a gravitational field depends on
(a) the end points only
(b) the path followed
(c) the velocity of the mass
(d) both length of the path and the end points

35. The escape velocity for a body projected vertically upwards from the surface of earth is 11 km sec^{-1}. If the body is projected at an angle of 45° with the vertical, the escape velocity will be
(a) $11/\sqrt{2}$ km sec^{-1} (b) $11\sqrt{2}$ km sec^{-1}
(c) 2 km sec^{-1} (d) 11 km sec^{-1}

36. The escape velocity corresponding to a planet of mass M and radius R is 50 km sec^{-1}. If the planet's mass and radius were $4M$ and R respectively, then the corresponding escape velocity would be
(a) 100 km sec^{-1} (b) 50 km sec^{-1}
(c) 200 km sec^{-1} (d) 25 km sec^{-1}

37. For a satellite moving in an orbit around the earth, the ratio of its potential energy to kinetic energy is
(a) 1 (b) –1 (c) 2 (d) –2

38. An astronaut experiences weightlessness in a space satellite. It is because
(a) the gravitational force is small at that location in space.
(b) the gravitational force is large at that location in space.
(c) the astronaut experiences no gravity.
(d) the gravitational force is infinitely large at that location in space.

39. The escape velocity from the earth is 11 km s^{-1}. The escape velocity from a planet having twice the radius and the same mean density as the earth would be
(a) 5.5 km s^{-1} (b) 11 km s^{-1}
(c) 15.5 km s^{-1} (d) 22 km s^{-1}

40. There is no atmosphere on moon as
(a) it is closer to earth.
(b) it revolves around the earth.
(c) it gets light from the sun.
(d) The rms speed of gas molecules is greater than the escape speed.

41. Two satellites of masses M and $4M$ are orbiting the earth in a circular orbit of radius r. Their frequencies of revolution are in the ratio of
(a) 1 : 4 (b) 4 : 1 (c) 1 : 2 (d) 1 : 1

42. The radius in kilometres, to which the present radius of the earth (R = 6400 km) to be compressed so that the escape velocity is increased 10 times, is
(a) 6.4 (b) 64 (c) 640 (d) 4800

43. The time period of an earth satellite in circular orbit is independent of
(a) the mass of the satellite
(b) radius of its orbit
(c) both the mass of satellite and radius of the orbit
(d) neither the mass of satellite nor the radius of its orbit

44. A satellite is launched into a circular orbit of radius R around earth while a second satellite is launched into an orbit of radius $1.02R$. The percentage difference in the time period is
(a) 0.7% (b) 1.0% (c) 1.5% (d) 3.0%

45. **Given below are two statements labelled as Assertion (A) and Reason (R)**
Assertion (A) : A body becomes weightless at the centre of earth.
Reason (R) : As the distance from centre of earth decreases, acceleration due to gravity increases.
Select the most appropriate answer from the options given below:
(a) Both A and R are true and R is the correct explanation of A.
(b) Both A and R are true but R is not the correct explanation of A.
(c) A is true but R is false.
(d) A is false and R is also false.

46. **Given below are two statements labelled as Assertion (A) and Reason (R)**
Assertion (A) : Gravitational field intensity is zero both at centre and infinity.
Reason (R) : The dimensions of gravitational field intensity is $[LT^{-2}]$.
Select the most appropriate answer from the options given below:
(a) Both A and R are true and R is the correct explanation of A.

(b) Both A and R are true but R is not the correct explanation of A.
(c) A is true but R is false.
(d) A is false and R is also false.

47. Given below are two statements labelled as Assertion (A) and Reason (R)

Assertion (A) : Smaller the orbit of the planet around the sun, shorter is the time it takes to complete one revolution.

Reason (R) : According to Kepler's third law of planetary motion, square of time period is proportional to cube of mean distance from sun.

Select the most appropriate answer from the options given below:
(a) Both A and R are true and R is the correct explanation of A.
(b) Both A and R are true but R is not the correct explanation of A.
(c) A is true but R is false.
(d) A is false and R is also false.

48. Given below are two statements labelled as Assertion (A) and Reason (R)

Assertion (A) : If earth suddenly stops rotating about its axis, then the value of acceleration due to gravity will become same at all the places.

Reason (R) : The value of acceleration due to gravity is independent of rotation of earth.

Select the most appropriate answer from the options given below:
(a) Both A and R are true and R is the correct explanation of A.
(b) Both A and R are true but R is not the correct explanation of A.
(c) A is true but R is false.
(d) A is false and R is also false.

49. Given below are two statements labelled as Assertion (A) and Reason (R)

Assertion (A) : There is no effect of rotation of earth on acceleration due to gravity at poles.

Reason (R) : Rotation of earth is about polar axis.

Select the most appropriate answer from the options given below:
(a) Both A and R are true and R is the correct explanation of A.
(b) Both A and R are true but R is not the correct explanation of A.
(c) A is true but R is false.
(d) A is false and R is also false.

SECTION-C

This section consists of 6 multiple choice questions with an overall choice to attempt any 5. In case more than desirable number of questions are attempted, ONLY first 5 will be considered for evaluation.

50. Infinite number of bodies, each of mass 2 kg are situated on x-axis at distances 1 m, 2 m, 4 m, 8 m, ..., respectively, from the origin. The resulting gravitational potential due to this system at the origin will be
(a) $-4/3G$ (b) $-4G$ (c) $-G$ (d) $-8/3G$

51. Gravitational potential of a body of mass m at a height h from the surface of earth of radius R is (Take g = acceleration due to gravity at earth's surface)
(a) $-g(R+h)$ (b) $-g(R-h)$
(c) $g(R+h)$ (d) $g(R-h)$

Case Study : Read the following paragraph and answer the questions :

Escape Speed

The escape speed on the earth (or any planet) is defined as the minimum speed with which a body has to be projected vertically upwards from the surface of earth (or any other planet) so that it just crosses the gravitational field of earth (or of that planet) and never returns on its own. Escape speed v_e is given by

$$v_e = \sqrt{\frac{2GM}{R}}$$

where, M = Mass of the earth/planet
R = Radius of the earth/planet

$$\text{or} \quad v_e = \sqrt{\frac{2G}{R} \times \frac{4}{3}\pi R^3 \rho} = \sqrt{\frac{8\pi\rho G R^2}{3}}$$

For earth, $v_e = 11.2 \text{ km s}^{-1}$

If a body is projected from a planet with a speed v which is smaller than the escape speed v_e (*i.e.* $v < v_e$), then the body will reach a certain height may either move in an orbit around the planet or may fall back to the planet.

If speed of projection (v) of the body from the surface of a planet is greater than the escape speed (v_e) of that planet, the body will escape out from the gravitational field of that planet and will move in the interstellar space with speed v' and v' is given by $v' = \sqrt{v^2 - v_e^2}$

52. On any planet, the presence of atmosphere implies that [C_{rms} = root mean square velocity of molecules and V_e = escape velocity]
(a) $C_{rms} < V_e$ (b) $C_{rms} > V_e$
(c) $C_{rms} = V_e$ (d) $C_{rms} = 0$

53. The mass of a planet is six times that of the earth. The radius of the planet is twice that of the earth. If the escape velocity from the earth is *ve*, then the escape velocity from the planet is

(a) $\sqrt{3}v_e$ (b) $\sqrt{2}v_e$ (c) v_e (d) $\sqrt{5}v_e$

54. Two planets A and B have the same material density. If the radius of A is twice the radius of B, then the ratio of their escape velocities (V_A/V_B) is

(a) 1/2 (b) 2 : 1 (c) $\sqrt{2}:1$ (d) $1:\sqrt{2}$

55. A body falls from infinity to the surface of the earth. What will be its velocity when it reaches the earth?

(a) $\sqrt{gR}$ (b) $\sqrt{2gR}$ (c) $2gR$ (d) gR

SOLUTIONS

1. **(a) :** According to Kepler's law, $T^2 \propto R^3$ or $T^2 = kR^3$ where k is a constant.
Taking logarithms on both sides,
$2 \log T = \log k + 3 \log R$
Differentiating, we get $2\left(\frac{\delta T}{T}\right)=0+3\left(\frac{\delta R}{R}\right)$

$$\therefore \frac{\delta T}{T}=\frac{3}{2}\left(\frac{\delta R}{R}\right)=\frac{3}{2}\left(\frac{1.01R-R}{R}\right)\times 100=1.5\%$$

2. **(c) :** Here, $M = 100$ kg, $m = 10$ g $= 10^{-2}$ kg
$R = 10$ cm $= 0.01$ m
Initial P.E. of the two bodies,

$$U_i=-\frac{GMm}{R}=-6.67\times 10^{-9}\text{ J}$$

When the particle is far away from the sphere, the P.E. of the system will be zero, $U_f = 0$
$\therefore\ W = U_f - U_i = +6.67 \times 10^{-9}$ J

3. **(c) :** $V_{e_1}=\sqrt{\frac{2GM_1}{R_1}}=\sqrt{\frac{2G\times\frac{4}{3}\pi R_1^3\times\rho_1}{R_1}}$

$$\therefore\ V_{e_1}=\frac{2\sqrt{2}R_1}{\sqrt{3}}\sqrt{\pi G\rho_1}\ ;\ V_{e_2}=\frac{2\sqrt{2}\times 2R_1}{\sqrt{3}}\times\sqrt{\pi G\frac{\rho_1}{2}}$$

$$\therefore\ \frac{V_{e_2}}{V_{e_1}}=2\sqrt{\frac{\rho_1}{2\rho_1}}=\frac{2}{\sqrt{2}}=\sqrt{2}\quad\therefore\ V_{e_2}=\sqrt{2}V_{e_1}$$

4. **(b) :** In planetary motion, the angular momentum conservation leads to the law of areas.

5. **(a) :** From Kepler's second law of planetary motion, the velocity of a planet is maximum when its distance from the sun is least.

6. **(b) :** Equal areas are swept in equal time.
Here t_1 is the time taken to move from C to D and t_2 is the time taken to move from A to B.
$\therefore\ t_1 = 2t_2$

7. **(b) :** A geostationary satellite goes around the earth in west-east direction.
The time period of a geostationary satellite is 24 hours.
The angle between the equatorial plane and the orbital plane of geostationary satellite is 0°.

8. **(b) :** $g=\frac{GM}{R^2}$

If both M and R decrease by 1%, their values become 0.99 M and 0.99 R respectively.

$$\therefore\ g'=\frac{G\times 0.99\,M}{(0.99R)^2}=1.01\frac{GM}{R^2}=1.01g$$

9. **(d) :** P.E.$_p$ = P.E.$_e$
$m\,g_p\,h_p = m\,g_e\,h_e$

$$h_p=\frac{g_e}{g_p}\times h_e \qquad \text{...(i)}$$

$$g_e=\frac{GM_e}{R_e^2}=\frac{G}{R_e^2}\times\left(\frac{4\pi}{3}R_e^3\rho_e\right)=\frac{4\pi}{3}GR_e\rho_e \qquad \text{...(ii)}$$

$$\therefore\ g_p=\frac{4\pi}{3}GR_p \qquad \text{...(iii)}$$

From equation (ii) and (iii), we get.

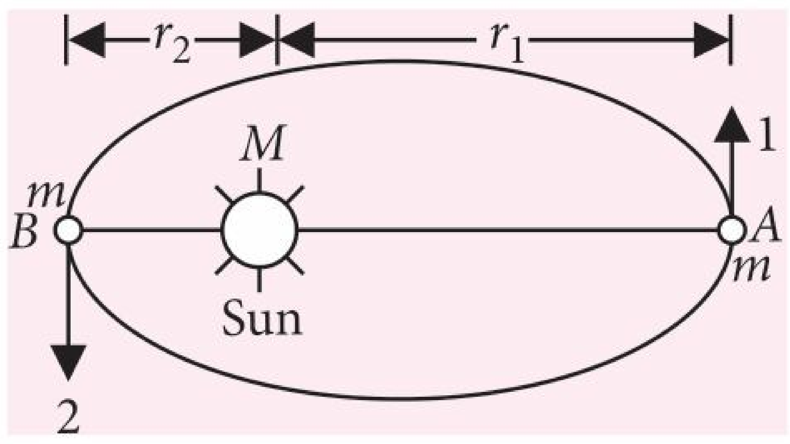
$$\frac{g_e}{g_p}=\frac{R_e\rho_e}{R_p\rho_p}=4\times 3=12$$

From equation (i), we have, $h_p = 12 \times 2 = 24$ m

10. **(d) :** Since radius vector sweeps equal areas in equal interval of time, area swept in one week = 1/4 × area swept in one month = $A/4$

11. **(c) :** According to Kepler's third law, $T^2 \propto r^3$, at $r = 0$, $T = 0$. It shows that the graph between T^2 and r^3 is a straight line passing through origin.

12. **(d) :** Using law of conservation of angular momentum, at locations A and B, we get,



$$L=mV_1r_1=mV_2r_2\Rightarrow V_2=\frac{V_1r_1}{r_2} \qquad \text{...(i)}$$

Using the principle of conservation of total energy at A and B,

$$\frac{1}{2}mV_1^2-\frac{GMm}{r_1}=\frac{1}{2}mV_2^2-\frac{GMm}{r_2}$$

$$\text{or}\quad V_2^2-V_1^2=2GM\left(\frac{1}{r_2}-\frac{1}{r_1}\right) \qquad \text{...(ii)}$$

Putting the values from Eq. (i) in Eq. (ii) and solving, we get, $V_1 = \left[\frac{2GMr_2}{r_1(r_1+r_2)}\right]^{1/2}$

$$\Rightarrow L = mV_1r_1 = m\left[\frac{2GMr_1r_2}{(r_1+r_2)}\right]^{1/2} = \left[\frac{2GMm^2r_2r_1}{r_1+r_2}\right]^{1/2}$$

13. (c) : From Kepler's law, $T^2 \propto R^3 \Rightarrow T \propto R^{3/2}$

$$\Rightarrow \frac{T'}{T} = \left(\frac{R'}{R}\right)^{3/2} \Rightarrow \frac{T'}{T} = \left(\frac{4R}{R}\right)^{3/2} = (4)^{3/2} = (2^2)^{3/2} = 2^3 = 8$$

$\therefore \; T' = 8T = 8 \times 90 = 720$ min

14. (c) : According to Kepler's third law,

$$T^2 \propto R^3 \Rightarrow \frac{T_2}{T_1} = \left(\frac{R_2}{R_1}\right)^{3/2}$$

$$\therefore \; \frac{T_2}{T_1} = \left(\frac{3R}{R}\right)^{3/2} = \sqrt{27} \quad \therefore \; T_2 = \sqrt{27}T_1 = 4\sqrt{27} \text{ h}$$

15. (c) : According to universal law of gravitation,

$$F = \frac{Gm_1m_2}{r^2} \quad \text{or} \quad F \propto \frac{1}{r^2}$$

Hence, it is also known as inverse square law.

16. (a) : According to Newton's law of gravitation

$$F \propto \frac{1}{d^2} \quad \text{or} \quad F \propto d^{-2}$$

17. (a) : For $r \le R$, $F = \frac{GM}{R^3}r$ or $F \propto r$

$$\therefore \; \frac{F_1}{F_2} = \frac{r_1}{r_2} \text{ if } r_1 < R \text{ and } r_2 < R$$

and for $r \ge R$, $F = \frac{GM}{r^2}$ or $F \propto \frac{1}{r^2}$

$$\therefore \; \frac{F_1}{F_2} = \frac{r_2^2}{r_1^2} \text{ if } r_1 > R \text{ and } r_2 > R$$

18. (b) : $g_h = g\left(1 - \frac{2h}{R}\right)$

$$\Delta g_1 = g - g_h = \frac{2hg}{R}; \; g_d = g\left(1 - \frac{h}{R}\right)$$

$$\Delta g_2 = g - g_d = \frac{gh}{R} \quad \therefore \; \Delta g_1 = 2\Delta g_2$$

19. (b) : Acceleration due to gravity at depth d is

$$g' = g\left(1 - \frac{d}{R}\right) = g\left(1 - \frac{32}{6400}\right) = \frac{199g}{200}$$

Therefore, decrease in weight is

$$mg - mg' = m\left(g - \frac{199g}{200}\right) = \frac{mg}{200}$$

$$\therefore \; \text{Percentage decrease} = \frac{mg/200}{mg} \times 100 = 0.5\%$$

20. (b) : $g_h = \frac{gR^2}{(R+h)^2}$ Given : $g_h = \frac{g}{100}$

$$\therefore \; \frac{gR^2}{(R+h)^2} = \frac{g}{100} \quad \text{or} \quad R + h = 10R \quad \text{or} \quad h = 9R$$

21. (b) : At a height h above the surface,

$$g_h = g\left(\frac{R}{R+h}\right)^2$$

Here, $g_h = \frac{a_g}{4}$ and $g = a_g$

$$\therefore \; \frac{a_g}{4} = a_g\left[\frac{R}{R+h}\right]^2 \quad \text{or} \quad \frac{R}{R+h} = \frac{1}{2}$$

or $R + h = 2R$ or $h = R$

$\therefore$ Distance from the centre = $R + h = R + R = 2R$

At a depth d below the surface, $g_d = g\left(1 - \frac{d}{R}\right)$

Here, $g_d = \frac{a_g}{4}$ and $g = a_g$

$$\therefore \; \frac{a_g}{4} = a_g\left(1 - \frac{d}{R}\right) \quad \text{or} \quad 1 - \frac{d}{R} = \frac{1}{4} \quad \text{or} \quad d = \frac{3}{4}R$$

$\therefore$ Distance from the centre = $R - d = R - \frac{3}{4}R = 0.25R$

Hence the required answer is $(2R, 0.25R)$.

22. (a) : $g_h = g\left(\frac{R}{R+h}\right)^2$ Given : $g_h = \frac{g}{2}$

$$\therefore \; \frac{g}{2} = g\left(\frac{R}{R+h}\right)^2 \quad \text{or} \quad \frac{1}{2} = \left(\frac{R}{R+h}\right)^2$$

or $R\sqrt{2} = (R+h)$ or $h = R(\sqrt{2} - 1)$

23. (b) : $g' = g\left[1 - \frac{R\omega^2}{g}\cos^2\lambda\right]$

At the equator, $\lambda = 0$

$$\therefore \; g_E = g\left[1 - \frac{R\omega^2}{g}\right] = g - R\omega^2$$

At the poles, $\lambda = 90°$

$\therefore \; g_p = g$

$\therefore \; g_p - g_E = g - g - (-R\omega^2) = R\omega^2$

24. (b) : The effective acceleration due to gravity at a certain latitude (λ) is given by

$g' = g_p - \omega^2 R\cos^2\lambda$ ...(i)

In this problem, $g' = g_p - \frac{3}{4}\omega^2 R$ ...(ii)

Comparing (i) and (ii), we get

$$\cos^2\lambda = \frac{3}{4} \Rightarrow \cos\lambda = \frac{\sqrt{3}}{2} \Rightarrow \lambda = 30°$$

25. (b) : According to Kepler's third law (law of periods), $T^2 \propto R^3$

where, T is time taken by the planet to go once around the sun R is semi-major axis (distance) of the elliptical orbit.

$\therefore\ T^2 = kR^3$ ...(i)

(where, k is constant of proportionality)

When R becomes 4 times, let time period be T'. Then,

$\therefore\ T'^2 = k(4R)^3$ ...(ii)

$$\therefore\ \frac{T^2}{T'^2} = \frac{1}{64} \text{ or } \frac{T}{T'} = \frac{1}{8} \text{ or } T' = 8T$$

So, time period becomes 8 times of previous value.

26. (c) : The acceleration due to gravity at a depth d from the surface of the earth is given by

$$g_d = g\left(1 - \frac{d}{R}\right)$$

Given: $g_d = \frac{g}{n}$ and $g_d = g\left(1 - \frac{d}{R}\right)$

$$\therefore\ \frac{g}{n} = g\left(1 - \frac{d}{R}\right) \text{ or } \frac{d}{R} = 1 - \frac{1}{n} = \frac{n-1}{n} \text{ or } d = \frac{R(n-1)}{n}$$

27. (b) : $g = \frac{GM}{R^2} = \frac{G}{R^2}\frac{4}{3}\pi R^3\rho = \frac{4}{3}\pi GR\rho\ \therefore\ \frac{g_1}{g_2} = \frac{R_1\rho_1}{R_2\rho_2}$

28. (d) : Given, $mg = 72 = \frac{GMm}{R^2}$

$$F' = mg' = \frac{GMm}{\left(R + \frac{R}{2}\right)^2} = \frac{GMm}{\left(\frac{3R}{2}\right)^2}$$

or $F' = \frac{4}{9}\frac{GMm}{R^2} = \frac{4}{9} \times 72 = 32$ N

29. (b) : The value of acceleration due to gravity at a height $h = \frac{64}{100}g$

$$\therefore\ \frac{64}{100}g = \frac{gR^2}{(R+h)^2} \text{ or } \frac{8}{10} = \frac{R}{R+h} \text{ or } h = \frac{R}{4}$$

30. (b) : G has different value in different system of units.

In SI system the value of G is 6.67×10^{-11} N m^2 kg^{-2} whereas in CGS its value is 6.67×10^{-8} dyne cm^2 g^{-2}.

The value of G is same throughout the universe.

The value of G was first experimentally determined by English scientist Henry Cavendish.

G is a scalar quantity.

31. (b) : Moon revolves around the earth in a nearly circular orbit. Sun exerts gravitational force on both, earth and moon. When observed from the sun, the orbit of the moon will not be strictly elliptical because the total gravitational force on it (*i.e.* force due to earth on moon and force due to sun on moon) is not central.

32. (b) : Here, mass of the earth (M) = 6.0×10^{24} kg, Mass of the body (m) = 50 kg and distance (r) = 6.3×10^9 m

Potential energy,

$$U = -\frac{GMm}{r} = -\frac{(6.7\times10^{-11})\times(6.0\times10^{24})\times50}{6.3\times10^9}$$

$= -3.19 \times 10^6$ J

33. (b) : It will remain the same as the gravitational force is independent of the medium separating the masses.

34. (a) : Gravitational force is a conservative force. Hence, work done in taking a mass from one point to another in a gravitational field depends on the end points only.

35. (d) : $v_e = \sqrt{\frac{2GM}{R}}$

Since the escape velocity is independent of direction of projection, therefore escape velocity of the body, projected at an angle of 45° with the vertical will be same, *i.e.*, 11 km sec^{-1}.

36. (a) : In first case, $v_e = \sqrt{\frac{2GM}{R}} = 50$

In second case, $v_e' = \left[\frac{2G(4M)}{R}\right]^{1/2} = 2\sqrt{\frac{2GM}{R}}$

$= 2 \times 50 = 100$ km sec^{-1}

37. (d) : Kinetic energy of the satellite, $K = \frac{GM_Em}{2r}$

Potential energy of the satellite, $U = -\frac{GM_Em}{r}$

Their corresponding ratio is $\frac{U}{K} = -2$

38. (c) : An astronaut experiences weightlessness in a space satellite. It is because the astronaut experiences no gravity.

39. (d) : Escape velocity from the earth is

$$v_e = \sqrt{\frac{2GM}{R}} = \sqrt{\frac{2G}{R}\frac{4}{3}\pi R^3\rho} = R\sqrt{\frac{8\pi}{3}G\rho}$$

As the planet has same density as that of earth,

$$\therefore\ \frac{v_p}{v_e} = \frac{2R}{R} = 2 \text{ or } v_p = 2v_e = 2\times11 = 22 \text{ km s}^{-1}$$

40. (d) : Moon has no atmosphere because the rms speed of gas molecules is greater than the escape speed (*i.e.*, $v_{rms} > ve$).

41. (d) : $T = 2\pi\sqrt{R^3/GM}$

The period and frequency of revolution are independent of the masses.

$\therefore$ The ratio is 1 : 1.

42. (b) : Escape velocity, $v_e = \sqrt{\frac{2GM}{R}}$ *i.e.* $v_e \propto \frac{1}{\sqrt{R}}$

where M and R are the mass and the radius of the earth.

$$\therefore \frac{v_{e_1}}{v_{e_2}} = \sqrt{\frac{R_2}{R_1}} \text{ or } \frac{1}{10} = \sqrt{\frac{R_2}{R_1}} \text{ or } \frac{1}{100} = \frac{R_2}{R_1}$$

$$\text{or } R_2 = \frac{R_1}{100} = \frac{6400}{100} = 64 \text{ km}$$

43. (a) : As $T^2 \propto r^3$ and proportionality constant does not depend on the mass of orbiting body. So, option (a) is correct.

44. (d) : According to Kepler's third law

$T^2 \propto R^3$ or $T \propto R^{3/2}$

$$\therefore \frac{T_2}{T_1} = \left(\frac{1.02R}{R}\right)^{3/2} = 1.03 \text{ or } T_2 = 1.03T_1$$

$$\therefore \% \text{ difference} = \frac{T_2 - T_1}{T_1} \times 100\% = 3\%$$

45. (c) : Variation of g with depth from surface of earth is given by $g' = g\left(1 - \frac{d}{R}\right)$.

At the centre of earth, $d = R$, $\therefore g' = g\left(1 - \frac{R}{R}\right) = 0$

$\therefore$ Apparent weight of body $= mg' = 0$

46. (b) : Gravitational field intensity at a point distance r from centre of earth is $E = \frac{GM}{r^2}$. When $r = \infty$, $E = 0$.

When point is inside the earth, then

$$E = \frac{G}{r^2} \times \frac{4}{3}\pi r^3 \rho = \frac{4\pi G\rho r}{3} \text{ when } r = 0, E = 0.$$

47. (a) : According to Kepler's third law of motion, the square of the time period of a planet about the sun is proportional to the cube of the semi major axis of the ellipse or mean distance of the planet from the sun. *i.e.* $T^2 \propto a^3$, when a is smaller, shorter is the time period.

48. (c) : The value of g at any place is given by the relation, $g' = g - \omega^2 Re \cos^2\lambda$

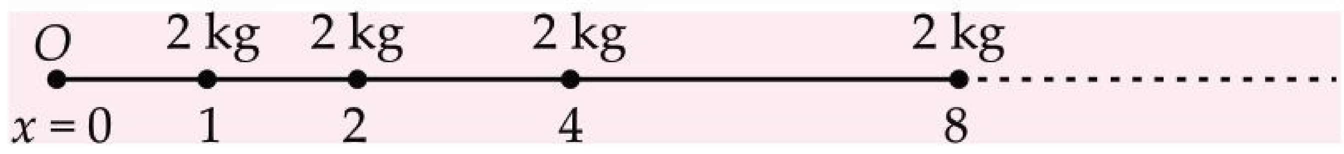
When λ is angle of latitude and ω is the angular velocity of earth. If $\omega = 0$,

$\therefore$ $g' = g$, if there is no rotation.

49. (a) : At poles, radius of horizontal circle is zero.

$\therefore$ Centripetal force $F = mr\omega^2 = 0$. Hence g at poles is not affected by rotation of earth.

50. (b) : The resulting gravitational potential at the origin O due to each of mass 2 kg located at positions as shown in figure is



$$V = -\frac{G\times 2}{1} - \frac{G\times 2}{2} - \frac{G\times 2}{4} - \frac{G\times 2}{8} - \ldots$$

$$= -2G\left[1 + \frac{1}{2} + \frac{1}{4} + \frac{1}{8} + \ldots\ldots\right] = -2G\left[\frac{1}{1 - \frac{1}{2}}\right]$$

$$= -2G\left[\frac{2}{1}\right] = -4G$$

51. (b) : Gravitational potential energy of the body at a height h is

$$U_h = -\frac{GMm}{(R+h)} = \frac{-gR^2m}{R\left(1+\frac{h}{R}\right)} = \frac{-gR^2m}{R}\left(1+\frac{h}{R}\right)^{-1}$$

$$\left(\because g = \frac{GM}{R^2}\right)$$

$$U_h = \frac{-gR^2m(R-h)}{R^2} = -gm(R-h)$$

$$\therefore V = \frac{U_h}{m} = \frac{-gm(R-h)}{m} = -g(R-h)$$

52. (a) : If $C_{rms} > V_e$, all the molecules will escape in space and there will be no atmosphere on the planet,

53. (a) : Escape velocity from the surface of earth is

$$v_e = \sqrt{\frac{2GM}{R}}$$

Escape velocity from the surface of planet is

$$v_p = \sqrt{\frac{2G(6M)}{(2R)}} = \sqrt{3}v_e$$

54. (b) : $\frac{V_{e_1}}{V_{e_2}} = \sqrt{\frac{M_1R_2}{M_2R_1}} = \sqrt{\frac{R_1^3}{R_2^3} \times \frac{R_2}{R_1}} = \frac{R_1}{R_2} = \frac{2}{1}$

55. (b) : The escape velocity for a body projected from the surface of the earth is $\sqrt{2gR}$. When we consider, V_e, we assume that the body is taken to an infinite distance from the surface of earth. Conversely if we bring a body from infinity to the surface of the earth it will acquire the velocity $V_e = \sqrt{2gR}$.

◈◈

CLASS-XII

BRUSH UP for NEET/JEE

Brush up your concepts to get high rank in NEET/JEE (Main and Advanced) by reading this column. This specially designed column is updated year after year by a panel of highly qualified teaching experts well-tuned to the requirements of these Entrance Tests.

Unit 3 Magnetic Effects of Current and Magnetism

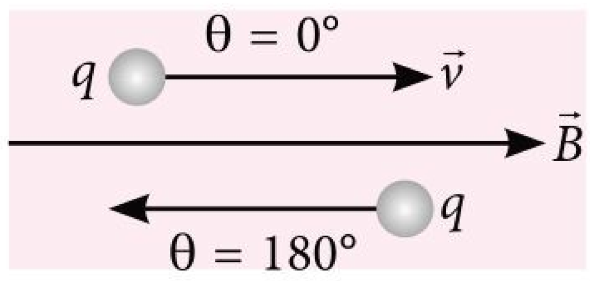
MAGNETIC FIELD

- The space surrounding a magnet where a magnetic force is experienced is called a magnetic field. The sources of magnetic fields are :
  - a current carrying conductor
  - changing electric field
  - moving charged particle
  - permanent magnet and electromagnet etc.

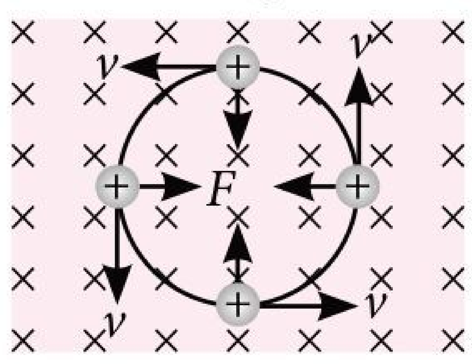
MAGNETIC FORCE ON A MOVING CHARGE

Trajectory of a Charged Particle in a Magnetic Field

- **Straight line :** If the direction of a particle moving with velocity $\vec{v}$ is parallel or antiparallel to magnetic field $\vec{B}$, *i.e.*, $\theta = 0$ or $\theta = 180°$ then $F = 0$. Hence the trajectory of the particle is a straight line.

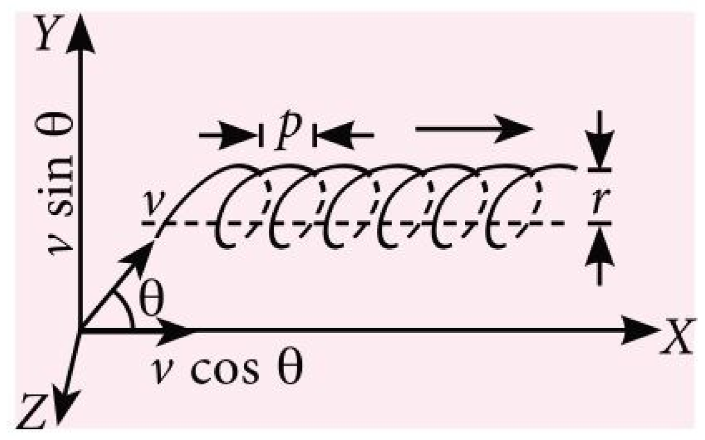


- **Circular path :** If $\vec{v}$ is perpendicular to $\vec{B}$, *i.e.*, $\theta = 90°$, then particle will experience a maximum magnetic force *i.e.*, $F_{max} = qvB$.



- Radius of path $r = \dfrac{mv}{qB} = \dfrac{p}{qB} = \dfrac{\sqrt{2mK}}{qB} = \dfrac{1}{B}\sqrt{\dfrac{2mV}{q}}$, where K is kinetic energy and V is potential.
- The time period of the particle $T = \dfrac{2\pi m}{qB}$

- **Helical path :** When the charged particle is moving at an angle to the field (other than 0°, 90° or 180°), particle describes a path called helix.



- Radius of path $r = \dfrac{m(v\sin\theta)}{qB}$
- Time period and frequency $T = \dfrac{2\pi m}{qB}$ and $\upsilon = \dfrac{qB}{2\pi m}$
- The pitch of the helix,

$$p = T(v\cos\theta) = 2\pi\frac{m}{qB}(v\cos\theta) = \frac{2\pi r}{\tan\theta}$$

Lorentz Force

- When the moving charged particle is subjected simultaneously to both electric field $\vec{E}$ and magnetic field $\vec{B}$, it experiences both electric force, $\vec{F}_e = q\vec{E}$ and magnetic force, $\vec{F}_m = q(\vec{v}\times\vec{B})$; so the net force on it will be $\vec{F} = q[\vec{E} + (\vec{v}\times\vec{B})]$.

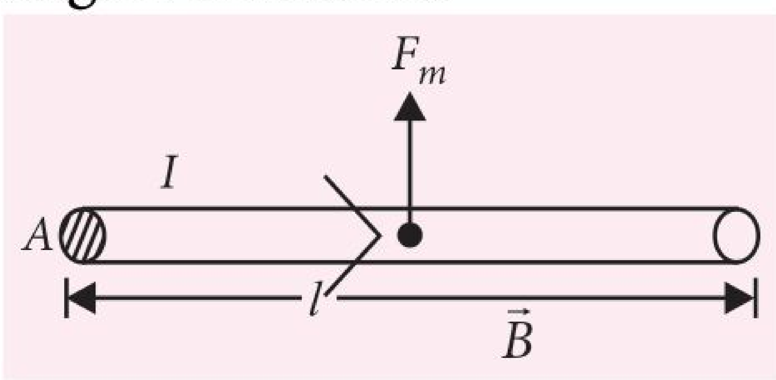
Cyclotron

- Cyclotron is a device used to accelerate positive (like, α-particles, deutrons etc.) to acquire enough energy to carry out nuclear disintegration etc.
- Cyclotron frequency : $\upsilon = \dfrac{1}{T} = \dfrac{Bq}{2\pi m}$
- Maximum energy of particle : $E_{max} = \left(\dfrac{q^2B^2}{2m}\right)r^2$

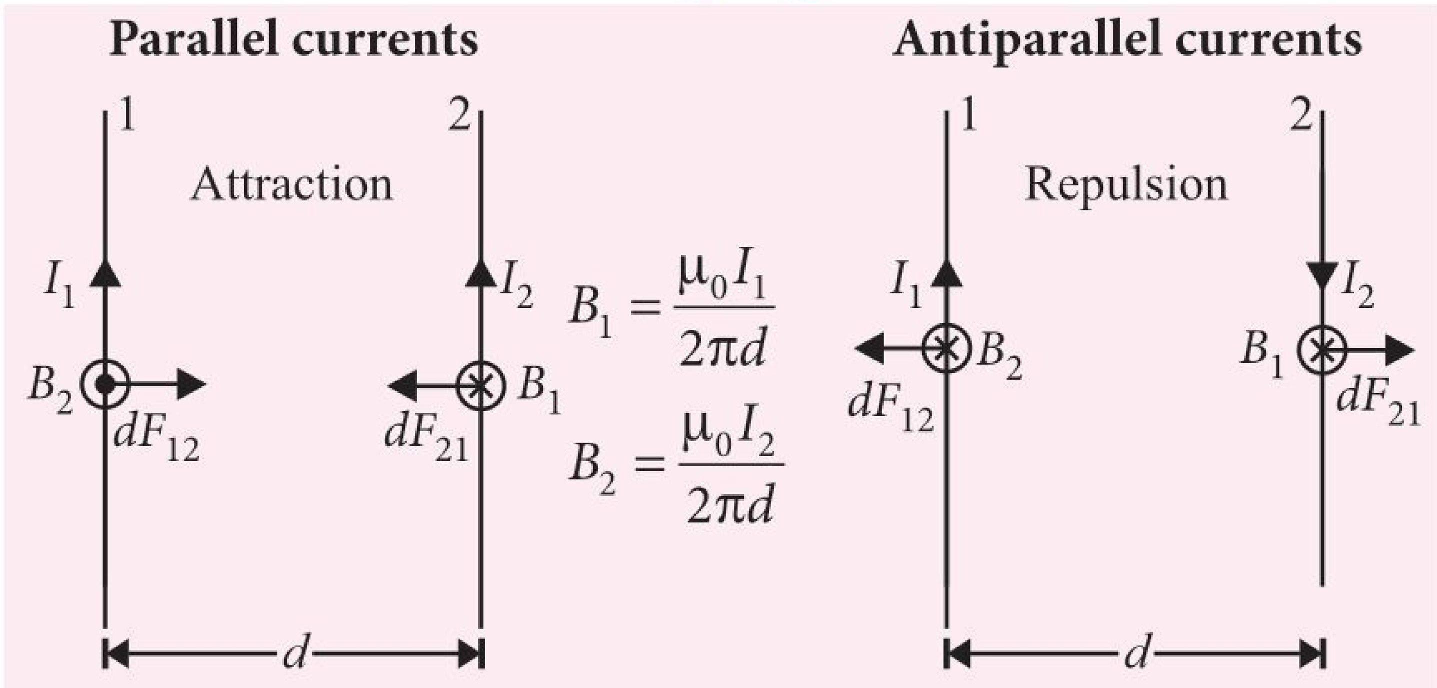
☞ Force on a Current Carrying Conductor

- Net mobile charge carriers in a conductor, $q = nAle$
- Average drift velocity of electron = v_d
- Magnetic force on charged conductor

$$\vec{F}_m = (nAl)e(\vec{v}_d \times \vec{B})$$
$$= [(ne\vec{v}_d)Al] \times \vec{B}$$
$$= (\vec{J}Al) \times \vec{B} = I(\vec{l} \times \vec{B})$$



☞ Magnetic Force between Two Long Parallel Current Carrying Wires

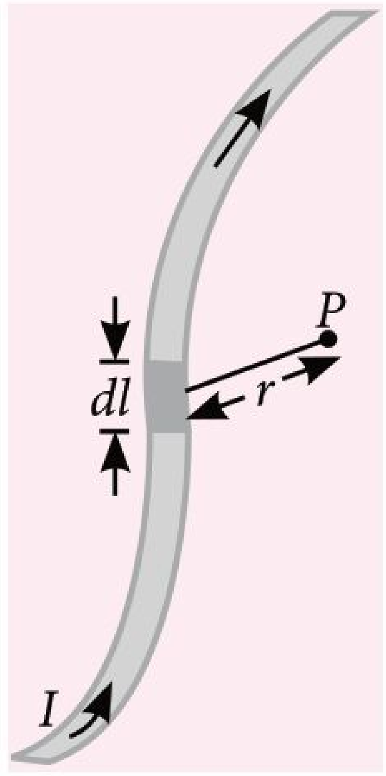


- Magnetic force per unit length of each wire is given as

$$\frac{dF_{12}}{dl} = \frac{dF_{21}}{dl} = \frac{\mu_0 I_1 I_2}{2\pi d}$$

Biot-Savart's Law

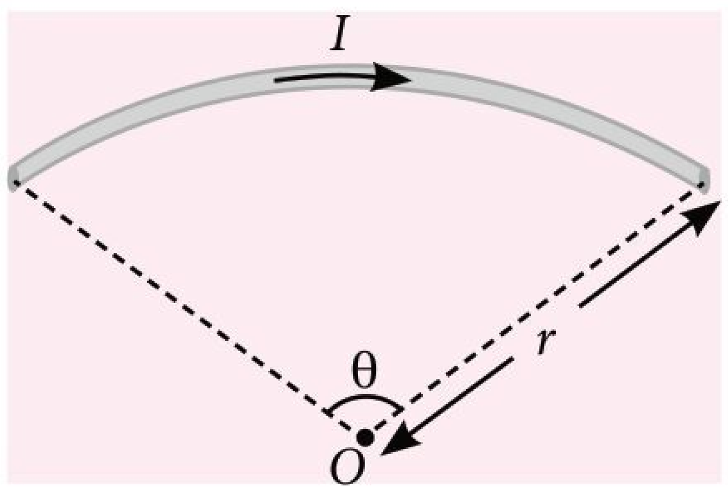
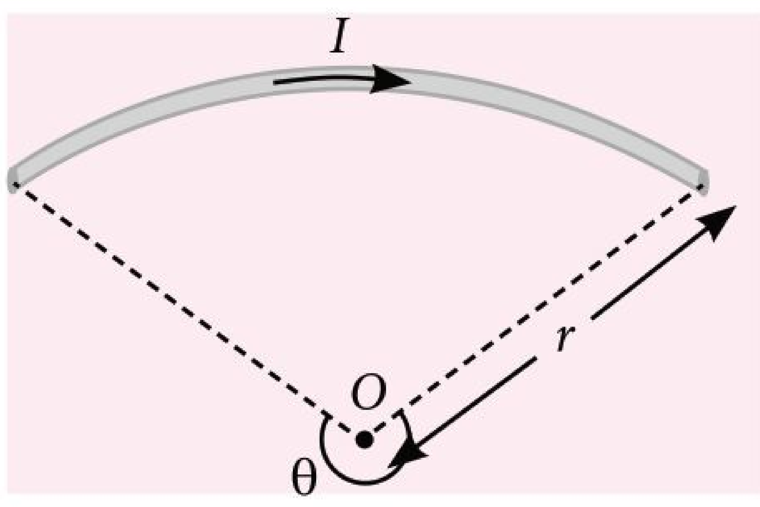
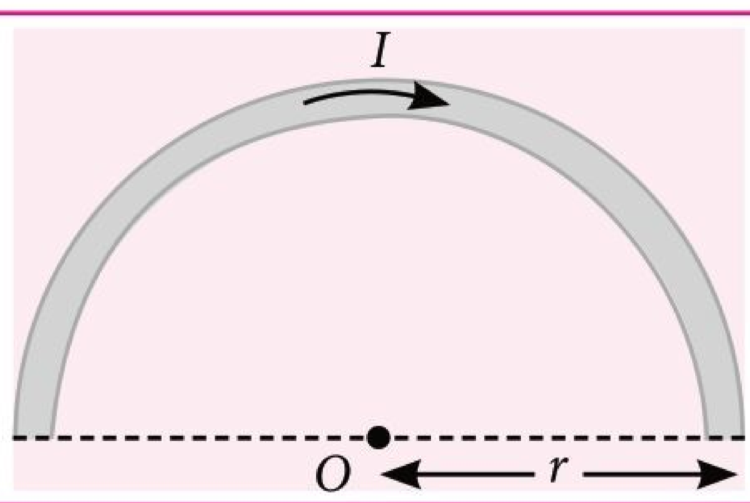
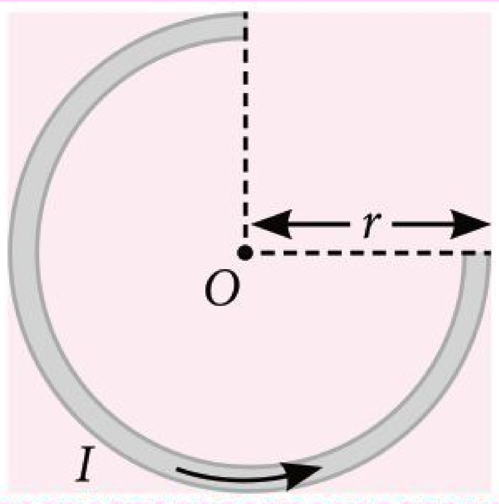
- Biot-Savart's law is used to determine the magnetic field at any point due to a current carrying conductor.
- According to Biot-Savart's law, magnetic field at point P due to the current element $I\vec{dl}$ is given by the expression $d\vec{B} = K\frac{I\,dl\sin\theta}{r^2}\hat{n}$

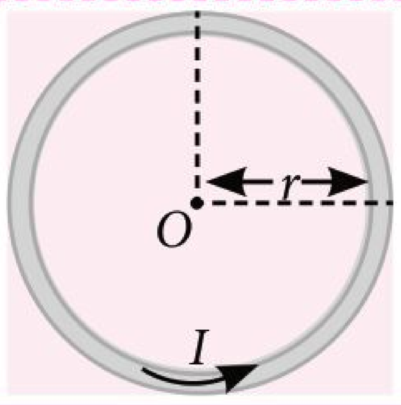
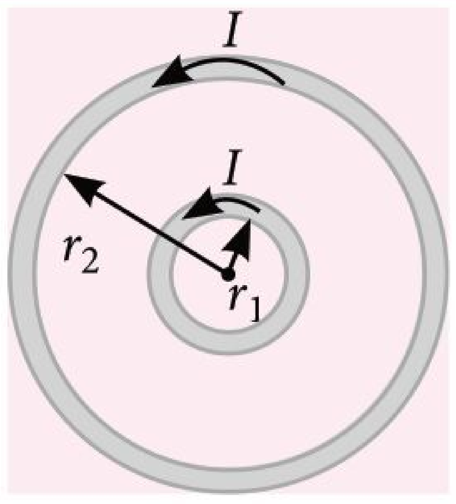
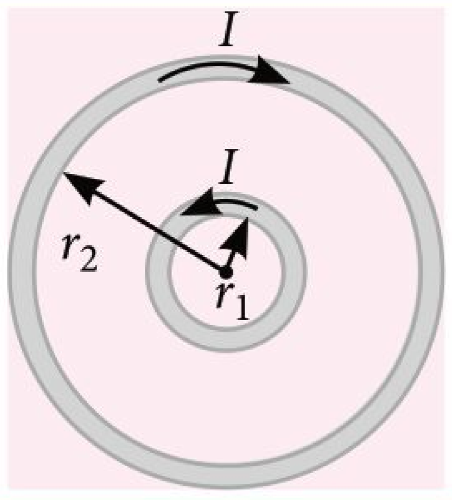
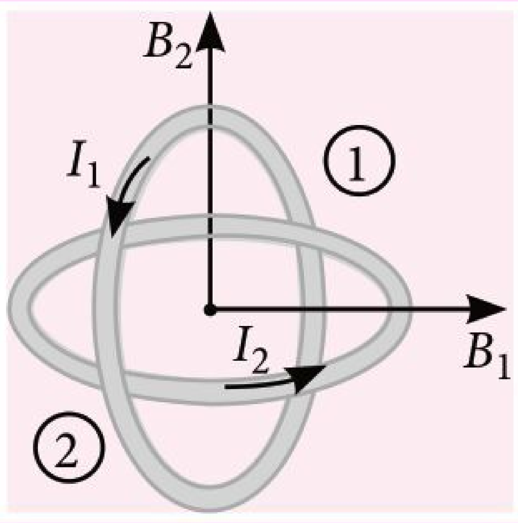
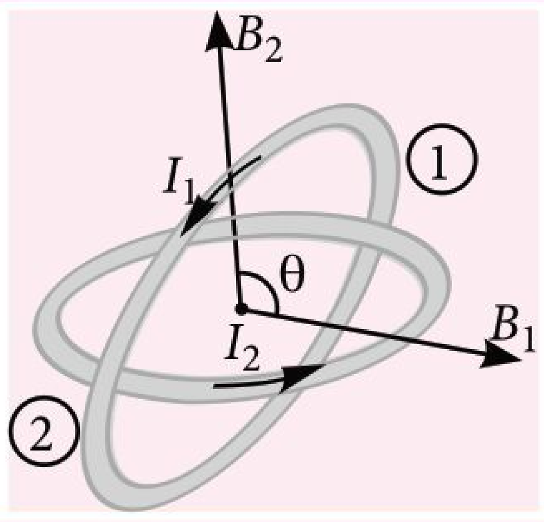
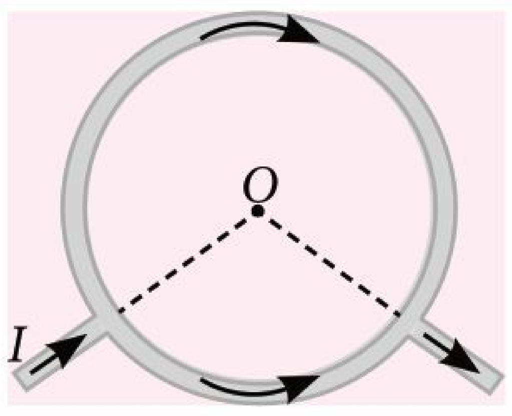


- Vectorially,

$$d\vec{B} = \frac{\mu_0}{4\pi}\cdot\frac{I(\vec{dl}\times\hat{r})}{r^2} = \frac{\mu_0}{4\pi}\cdot\frac{I\vec{dl}\times\vec{r}}{r^3}$$

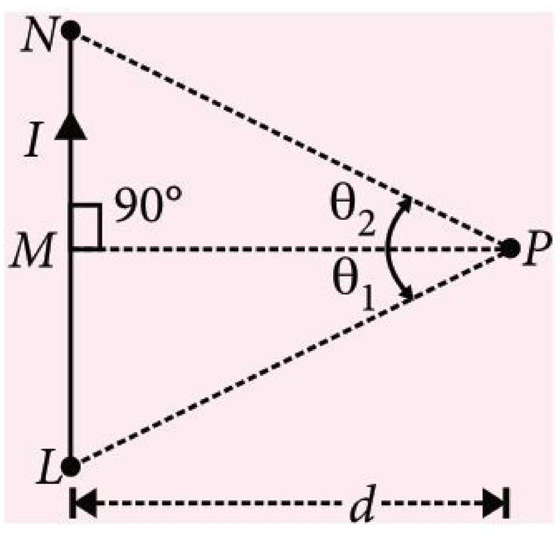
☞ Application of Biot-Savart's Law

Condition	Figure	Magnetic field at the centre
Arc subtends angle θ at the centre		$B = \frac{\mu_0}{4\pi}\cdot\frac{\theta I}{r}$
Arc subtends angle (2π – θ) at the centre		$B = \frac{\mu_0}{4\pi}\cdot\frac{(2\pi-\theta)I}{r}$
Semi-circular arc		$B = \frac{\mu_0}{4\pi}\cdot\frac{\pi I}{r} = \frac{\mu_0 I}{4r}$
Three quarter semi-circular current carrying arc		$B = \frac{\mu_0}{4\pi}\cdot\frac{\left(2\pi-\frac{\pi}{2}\right)I}{r} = \frac{3\mu_0 I}{8r}$

Circular current carrying arc		$B = \frac{\mu_0}{4\pi}\frac{2\pi I}{r} = \frac{\mu_0 I}{2r}$
Concentric coplanar circular loops carrying current in the same direction		(i) If the number of turns is same $B_1 = \frac{\mu_0}{2} I\left[\frac{1}{r_1}+\frac{1}{r_2}\right]\cdot n$ (ii) If number of turns are different $B_1 = \frac{\mu_0 I}{2}\left[\frac{n_1}{r_1}+\frac{n_2}{r_2}\right]$
Concentric coplanar circular loops carrying current in the opposite direction		(i) If the number of turns is same $B_2 = \frac{\mu_0}{2} I\left[\frac{1}{r_1}-\frac{1}{r_2}\right]\cdot n$ (ii) If number of turns are different $B_2 = \frac{\mu_0 I}{2}\left[\frac{n_1}{r_1}-\frac{n_2}{r_2}\right]$
Concentric loops but their planes are perpendicular to each other		$B = \sqrt{B_1^2 + B_2^2}$ $= \frac{\mu_0}{2r}\sqrt{I_1^2 + I_2^2}$
Concentric loops but their planes are at an angle θ with each other		$B = \sqrt{B_1^2 + B_2^2 + 2B_1B_2\cos\theta}$ $= \frac{\mu_0}{2r}\sqrt{I_1^2 + I_2^2 + 2I_1I_2\cos\theta}$
Distribution of current between any two points on the circumference		$B = 0$

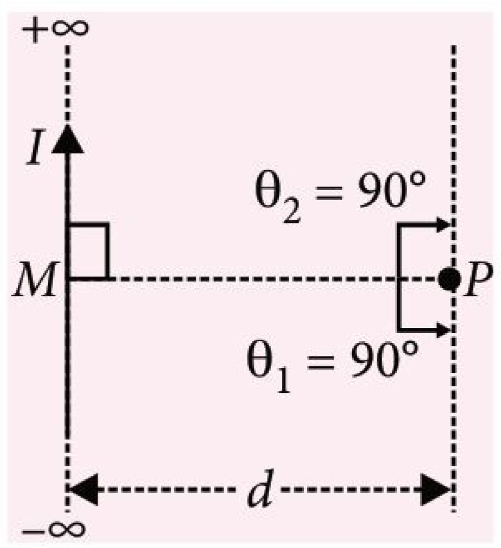
- **Magnetic field due to current carrying straight conductor or wire at point *P***

Finite length wire : $B_P = \frac{\mu_0 I}{4\pi d}(\sin\theta_1 + \sin\theta_2)$



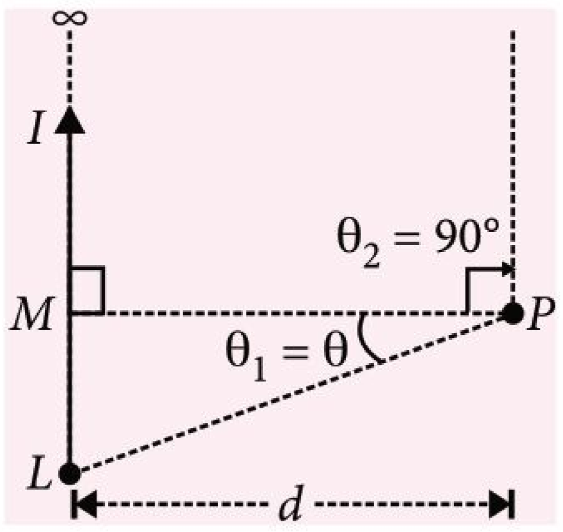
➢ Infinite length wire :

$$B_P = \frac{\mu_0 I}{4\pi d}(\sin 90° + \sin 90°)\ ;\ B_P = \frac{\mu_0 I}{2\pi d}$$



➢ Semi infinite length wire:

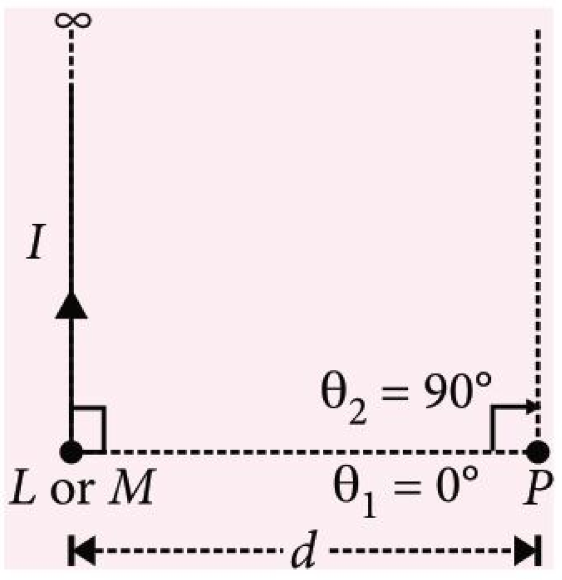
$$B_P=\frac{\mu_0 I}{4\pi d}(\sin\theta+\sin 90°)\ ;\ B_P=\frac{\mu_0 I}{4\pi d}(\sin\theta+1)$$



➤ Special semi infinite wire :

$$B_P=\frac{\mu_0 I}{4\pi d}(\sin 0°+\sin 90°)$$

$$B_P=\frac{\mu_0 I}{4\pi d}$$

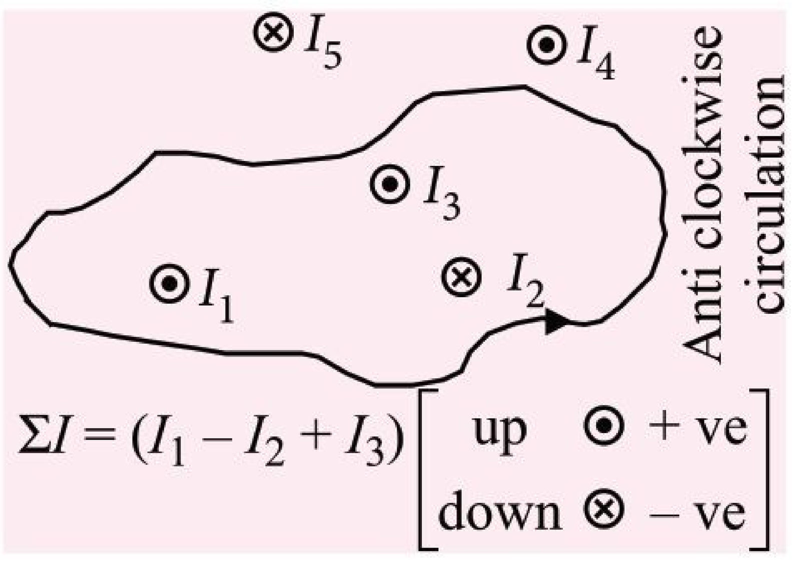


Ampere's Circuital Law

- It states that line integral of the magnetic field along any closed path in free space is equal to μ_0 times of net current, which crossing through area bounded by the closed path.

  Mathematically

  $$\oint \vec{B}\cdot d\vec{l}=\mu_0\,\Sigma I$$



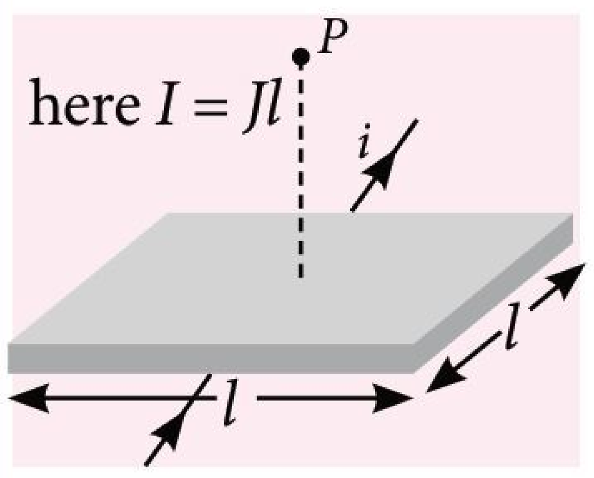
☞ Application of Ampere's Law

Magnetic field due to an infinite sheet carrying current

- An infinite sheet of current with linear current density J

  According to Ampere's law,

  $2Bl=\mu_0(Jl)$ or $B=\frac{\mu_0 J}{2}$

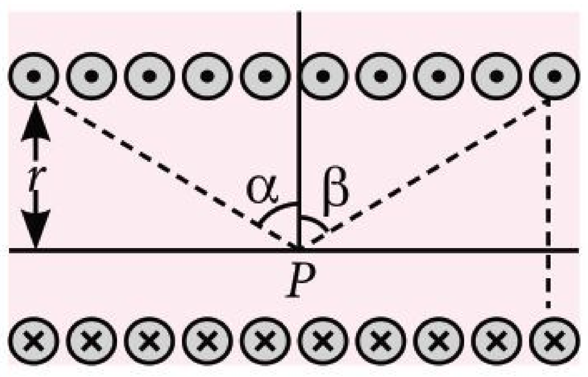


Solenoid

- Finite length solenoid :

$$B=\frac{\mu_0}{4\pi}(2\pi nI)[\sin\alpha+\sin\beta]$$

- Infinite length solenoid : If the solenoid is of infinite length *i.e.*, $\alpha=\beta=(\pi/2)$. So $B_{in}=\mu_0 nI$.



- If the solenoid is of infinite length and the point is near one end *i.e.*, $\alpha=0$ and $\beta=(\pi/2)$ so

  $$B_{end}=\frac{1}{2}(\mu_0 nI)$$

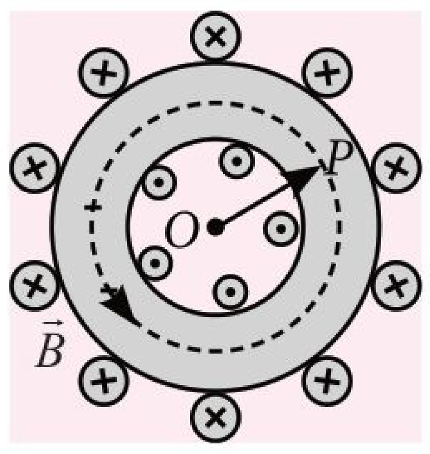
  $$\left(B_{end}=\frac{1}{2}B_{in}\right)$$

Toroid

A toroid can be considered as a ring shaped closed solenoid.

$$B=\frac{\mu_0 NI}{2\pi r}=\mu_0 nI,$$

where $n = N/2\pi r$



Torque on a current Loop

☞ Torque on a Rectangular Current Loop

- $\vec{\tau}=\vec{m}\times\vec{B}=mB\sin\theta\hat{n},$

  where, $\vec{m}=NI\vec{A}=NI(ab)\breve{A}$

  = magnetic moment of current in loop,

  $A = ab$ = area of rectangular loop ;

  θ = angle between $\vec{m}$ and $\vec{B}$

☞ Circular Current Loop as a Magnetic Dipole

- Magnetic moment of circular current loop, $m = IA = I\pi R^2$
- Magnetic field perpendicular to the loop at any point at a distance x from the centre, for $(x >> R)$,

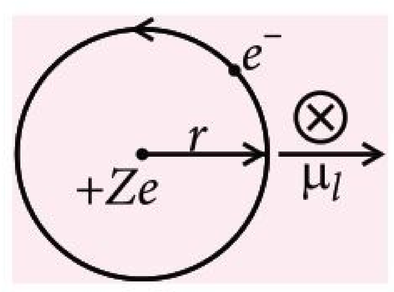
  $$B_\perp=\frac{\mu_0 I R^2}{2x^3}=\frac{\mu_0}{4\pi}\cdot\frac{2m}{x^3}$$

- Magnetic field in the plane of the loop at any point at a distance x from the centre, (for $x >> R$),

  $$B_{||}=\frac{\mu_0}{4\pi}\frac{m}{x^3}.\ \text{Also, } B_\perp=2\times B_{||}$$

☞ Magnetic Dipole Moment of a Revolving Electron

- Orbital current, $I=\frac{e}{T}=\frac{ev}{2\pi r}$
- Magnetic moment associated with the circulating current, $\mu_l=I\pi r^2=\frac{evr}{2}=\frac{e}{2m_e}L,$
- Vectorially, $\vec{\mu}_l=-\frac{e}{2m_e}\vec{L}$
- Gyromagnetic ratio, $\frac{\mu_l}{L}=\frac{e}{2m_e}$

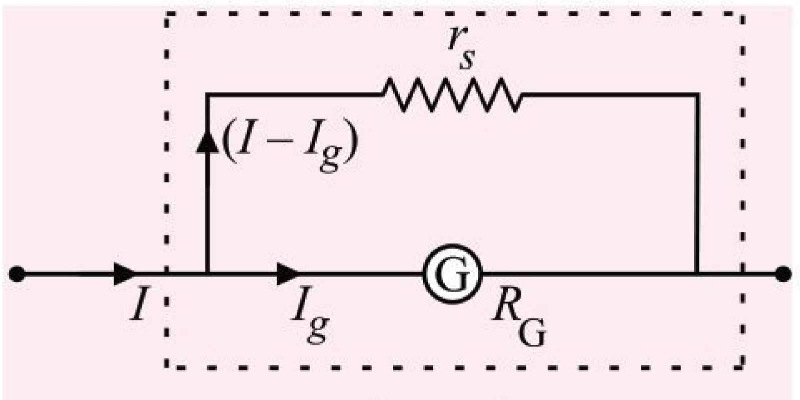


Key Point

- Bohr magneton, $\mu_B = (\mu_l)_{min} = \frac{e}{4\pi m_e}h$ $= 9.27 \times 10^{-24}$ A m^2
- If the plane of the coil is perpendicular to the direction of magnetic field *i.e.*, $\theta = 0°$, then $t = 0$ (minimum)
- If the plane of the coil is parallel to the direction of magnetic field *i.e.*, $\theta = 90°$, then t = NIAB (maximum)

Moving Coil Galvanometer

- **Principle :** When a current carrying coil is placed in a magnetic field, it experiences a torque. In moving coil galvanometer, the current I passing through the galvanometer is directly proportional to its deflection (ϕ).
  $I \propto \phi$ or $I = G\phi$,
  where $G = \frac{k}{NAB}$ = galvanometer constant
  A = area of the coil, N = number of turns in the coil, B = strength of magnetic field, k = torsional constant of the spring *i.e.*, restoring torque per unit twist.
- Current sensitivity, $\frac{\phi}{I} = \frac{NAB}{k}$
- Voltage sensitivity, $\frac{\phi}{V} = \left(\frac{NAB}{k}\right)\frac{I}{V} = \left(\frac{NAB}{k}\right)\frac{1}{R}$
- Conversion of galvanometer into ammeter :
  r_s is a very small resistance ($r_s << R_G$)

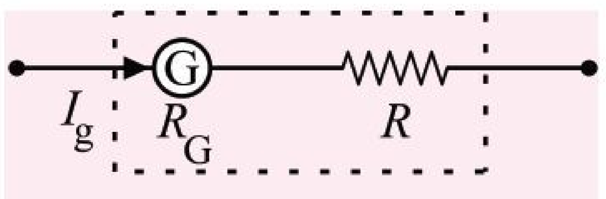
  

  Ammeter

  $\frac{r_s}{R_G} = \frac{I_g}{I - I_g}$

  $\therefore\ r_s = \left(\frac{I_g}{I - I_g}\right)R_G$

  (Shunt (r_s) is connected in parallel)
- Conversion of galvanometer into voltmeter : R is a large resistance ($R >> R_G$)
  $V = I_g(R_G + R)$
  $\therefore\ R = \frac{V}{I_g} - R_G$

  Voltmeter

  (A large resistance (R) is connected in series)

MAGNETISM

Various Term Related to Magnetism

Magnetic field and magnetic lines of force	Space around a magnetic pole or magnet or current carrying wire within which it's effect can be experienced.
Magnetic flux (ϕ) and flux density (B)	The number of magnetic lines of force passing normally through a surface is defined as magnetic flux (ϕ). S.I. unit : weber (Wb) and CGS unit is maxwell.
Magnetic permeability	It is the degree or extent to which magnetic lines of force can enter a substance and is denoted by μ.
Intensity of magnetising field ($\vec{H}$)	It is the degree of extent to which a magnetic field can magnetise a substance. Also $H = \frac{B}{\mu}$ SI unit : $\frac{\text{J}}{\text{m} \times \text{Wb}}$ or J m^{-1} Wb^{-1}
Intensity of magnetisation (I)	It can also be defined as the pole strength per unit cross sectional area of the substance or the induced dipole moment per unit volume. $I = \frac{m}{A} = \frac{M}{V}$ S.I. unit is A m^{-1}.
Magnetic susceptibility (χ_m)	It is the property of the substance which shows how easily a substance can be magnetised. $\chi_m = \frac{I}{H}$.
Relation between permeability and susceptibility	$B = B_0 + B_m = \mu_0 H + \mu_0 I$ $= \mu_0(H + I) = \mu_0 H(1 + \chi_m) = \mu_r H$ where $\mu_r = (1 + \chi_m)$.

MAGNETIC DIPOLE

- A magnetic dipole consists of two unlike poles of equal strength and separated by a small distance *e.g.*, a bar magnet, a compass needle etc. are magnetic dipoles.

Magnetic Dipole Moment

- It is defined as the product of strength of either pole (m) and the magnetic length $(2\vec{l})$ of the magnet. It is denoted by $\vec{M}$.
  Magnetic dipole moment = strength of either pole × magnetic length
  $\vec{M} = m(2\vec{l})$
- Magnetic dipole moment is a vector quantity and it is directed from south to north pole of the magnet.
- The SI unit of magnetic dipole moment is A m^2.

☞ Magnetic Field at a Point due to Magnetic Dipole (or Bar Magnet)

- The magnetic field due to a bar magnet at any point on the axial line (end on position) is given by

$$B_{\text{axial}} = \frac{\mu_0}{4\pi}\frac{2Mr}{(r^2 - l^2)^2}$$

where r = distance between the centre of the magnet and the given point on the axial line, $2l$ = magnetic length of the magnet and M = magnetic moment of the magnet.

For short magnet $l^2 << r^2$; $B_{\text{axial}} = \frac{\mu_0 2M}{4\pi r^3}$

The direction of B_{axial} is along S-N.

- The magnetic field due to a bar magnet at any point on the equatorial line (board-side on position) of the bar magnet is given by

$$B_{\text{equatorial}} = \frac{\mu_0 M}{4\pi(r^2 + l^2)^{3/2}}$$

For short magnet $l^2 << r^2$; $B_{\text{equatorial}} = \frac{\mu_0 M}{4\pi r^3}$

The direction of $B_{\text{equatorial}}$ is parallel to N-S.

☞ Current Loop as a Magnetic Dipole

- A current loop behaves as a magnetic dipole whose magnetic dipole moment is $M = IA$ where A is the area enclosed by loop and I is the current flowing in the loop.
- If there are N turns in a loop, then $M = NIA$.

☞ Magnetic Dipole Moment of a Revolving Electron

- An electron revolving around the nucleus has a magnetic moment μ_L and is given by
- $\mu_L = \frac{e}{2m}L$

where L is the magnitude of the angular momentum of the revolving electron around the nucleus. The smallest value of μ_L is called Bohr magneton μ_B and it is given by

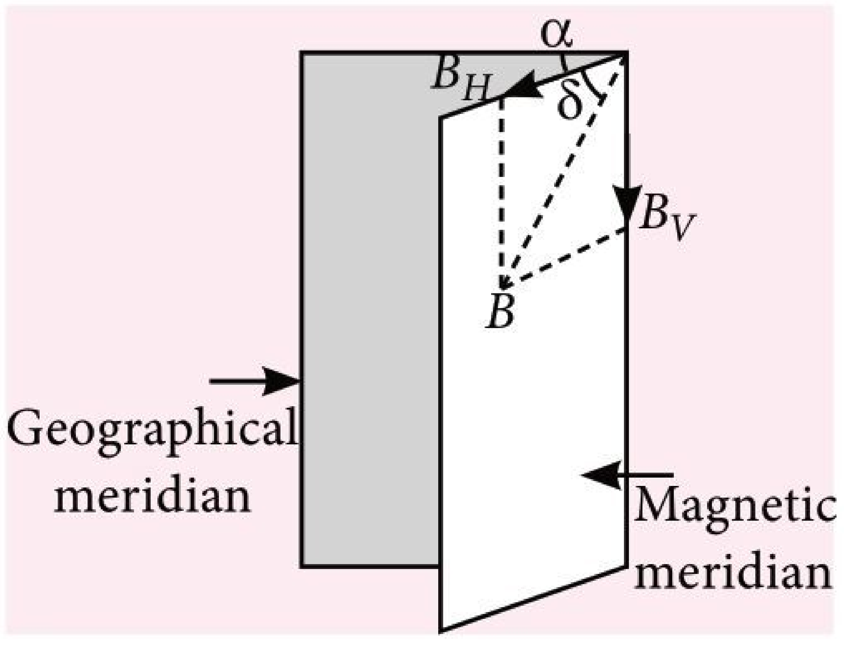
$$\mu_B = \frac{e\hbar}{2m} = 9.274\times10^{-24}\ \text{J T}^{-1} = 5.788\times10^{-5}\ \text{eV T}^{-1}.$$

where m is the mass of the electron and $\hbar = \frac{h}{2\pi}$.

Earth's Magnetism

☞ Magnetic Declination (α)

- It is the angle between geographic and the magnetic meridian planes.



☞ Angle of Inclination or Dip (δ)

- It is the angle between the direction of intensity of total magnetic field of earth and a horizontal line in the magnetic meridian.

☞ Earth's Magnetic Field

- $B_H = B\cos\delta$ ...(i) and $B_V = B\sin\delta$ ...(ii)

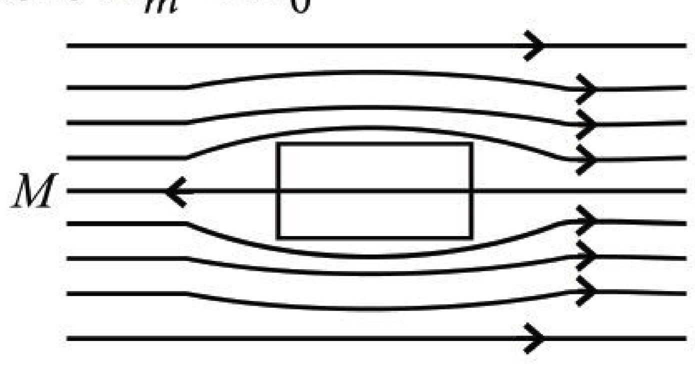
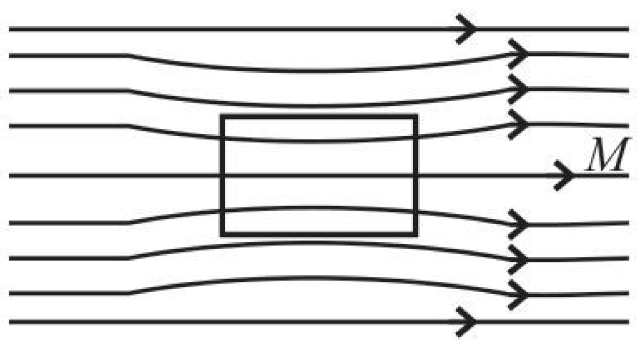
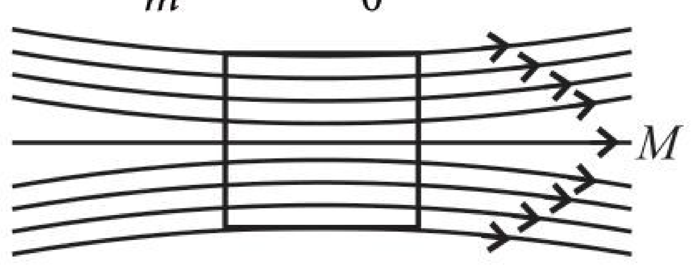
By squaring and adding equations (i) and (ii),

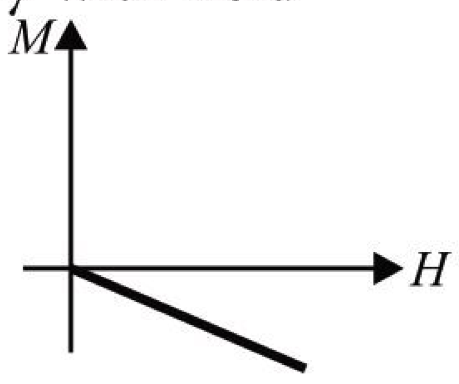
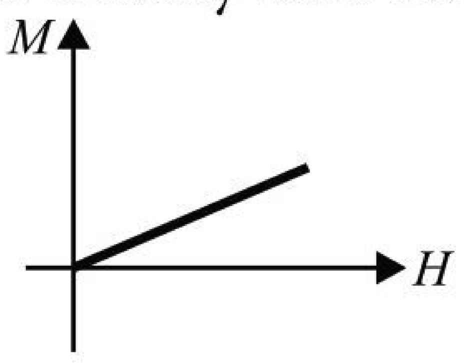
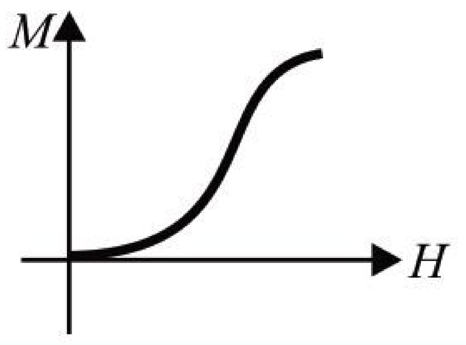
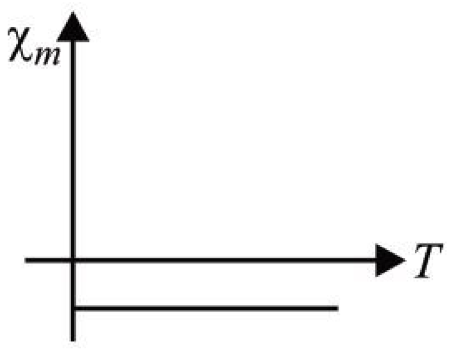
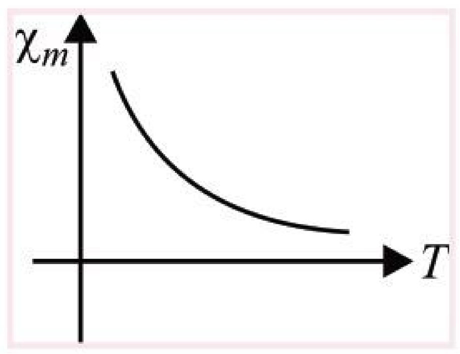
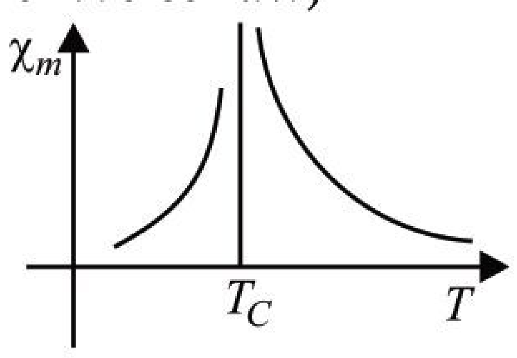
$$B = \sqrt{B_H^2 + B_V^2}$$

Dividing equation (ii) by equation (i),

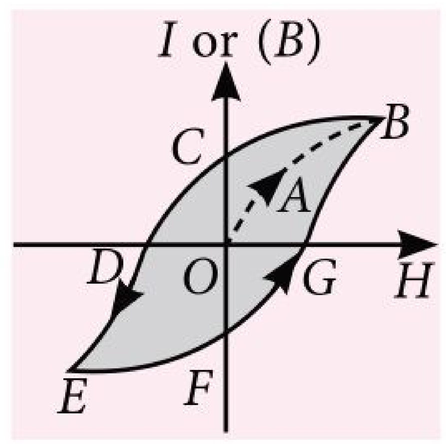
$$\tan\phi = \frac{B_V}{B_H}$$

Classification of Magnetic Materials

Properties	Diamagnetic	Paramagnetic	Ferromagnetic
Cause of magnetism	Orbital motion of electrons	Spin motion of electrons	Formation of domains
Substance placed in uniform magnetic field.	Poor magnetisation in opposite direction. Here $B_m < B_0$ 	Poor magnetisation in same direction. Here $B_m > B_0$ 	Strong magnetisation in same direction Here $B_m >>> B_0$ 

$M - H$ curve	$M \rightarrow$ Small, negative, varies linearly with field 	$M \rightarrow$ Small, positive, varies linearly with field 	$M \rightarrow$ Very large, positive and varies non-linearly with field 
$\chi_m - T$ curve	$\chi_m \rightarrow$ Small, negative and temperature independent $\chi_m \propto T^0$ 	$\chi_m \rightarrow$ Small, positive and varies inversely with temperature $\chi_m \propto \frac{1}{T}$ (Curie law) 	$\chi_m \rightarrow$ Very large, positive and temperature dependent $\chi_m \propto \frac{1}{T - T_C}$ (for $T > T_C$) (Curie-Weiss law)  $T_{C\text{ (iron)}}$ = 770 °C or 1043 K
μ_r (Relative Permeability)	$1 > \mu_r > 0$ ($\mu < \mu_0$)	$1 + \varepsilon > \mu_r > 1$ ($\mu > \mu_0$)	$\mu_r >>> 1$ ($\mu >>> \mu_0$)

Magnetic Hysteresis



- The lack of retracibility as shown in figure is called hysteresis and the curve is known as hysteresis loop.
- **Retentivity :** The property by virtue of which the magnetism (I) remains in a material even on the removal of magnetising field is called retentivity or residual magnetism.
- **Coercivity or coercive force :** When magnetic field H is reversed, the magnetisation decreases and for a particular value of H, denoted by H_c, it becomes zero *i.e.*, $H_c = OD$ when $I = 0$. This value of H is called the coercivity.

Magnetic hard substance (steel) $\rightarrow$ High coercivity
Magnetic soft substance (soft iron) $\rightarrow$ Low coercivity

Key Point

- Gauss law for magnetism states that the net magnetic flux through any closed surface is zero. *i.e.*, $\phi_B = 0$.
- If a magnet of moment M and pole strength m is cut into two equal parts along its length, then the pole strength of each part is $m/2$ and the magnetic moment of each part is $M/2$.
- If a magnet of magnetic moment M and pole strength m is cut into two equal halves along perpendicular to its length, then the pole strength of each part is m and magnetic moment of each part is $M/2$.

WRAP it up!

1. A straight wire of length π^2 metre carrying a current of 2 A and the magnetic field due to it is measured at a point distant 1 cm from it. If the wire were bent into a circle and were to carry the same current as before, the ratio of the magnetic field at its centre to that obtained in the first case would be
(a) 50 : 1 (b) 1 : 50
(c) 100 : 1 (d) 1 : 100

2. A bar magnet has a magnetic moment equal to 5×10^{-5} Wb m. It is suspended in a magnetic field which has a magnetic induction B equal to $8\pi \times 10^{-4}$ T. The magnet vibrates with a period of vibration equal to 15 s. The moment of inertia of magnet is
(a) 4.54×10^4 kg m^2 (b) 4.54×10^{-5} kg m^2
(c) 4.54×10^{-4} kg m^2 (d) 4.54×10^5 kg m^2

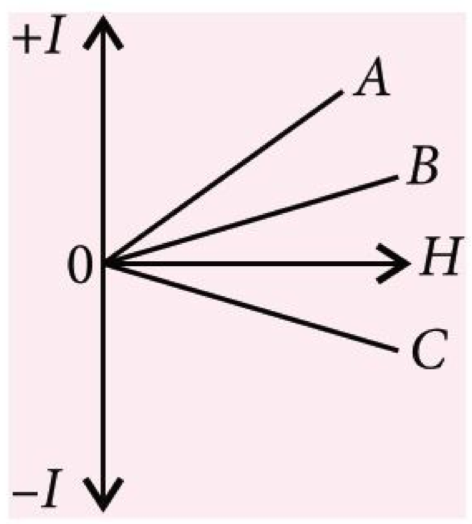
3. An element $\Delta\vec{l} = \Delta x\hat{i}$ is placed at the origin and carries a large current I = 10 A. What is the magnetic field on the y-axis at a distance of 0.5 m if Δx = 1 cm?
(a) 4×10^{-5} T along x-axis
(b) 8×10^{-8} T along z-axis
(c) 4×10^{-8} T along z-axis
(d) 8×10^{-6} T along y-axis

4. In a hydrogen atom, the electrons moves in a circular orbit of radius 0.5 Å, about the nucleus. What is the magnetic induction produced at the centre of the orbit, if the electrons makes 6.25×10^{15} rev per sec? [$e = 1.6 \times 10^{-19}$ C]
(a) 6.28 Wb m^{-2} (b) 9 Wb m^{-2}
(c) 12.56 Wb m^{-2} (d) 15 Wb m^{-2}

5. There is a uniform electric field of strength 10^3 V m^{-1} along the y-axis. A body of mass 1 g and charge 10^{-6} C is projected into the field from the origin along the positive x-axis with a velocity of 10 m s^{-1}. Its speed (in m s^{-1}) after 10 s will be (negative gravitation)
(a) 10 (b) $5\sqrt{2}$ (c) $10\sqrt{2}$ (d) 20

6. A solenoid of length 0.4 m and having 500 turns of wire carries a current of 3.0 A. A thin coil having 10 turns of wire and of radius 0.01 m carries a current of 0.04 A. What is the torque required to hold the coil in the middle of the solenoid with its axis perpendicular to the axis of the solenoid ? (Use $\pi^2 = 10$)
(a) 6×10^6 N m (b) 6×10^{-6} N m
(c) 7.5×10^{-6} N m (d) 4.2×10^{-6} N m

7. The given figure shows the variation of intensity of magnetization (I) versus the applied magnetic field intensity (H) for 3 magnetic materials A, B and C. Name the diamagnetic (D), ferromagnetic (F) and paramagnetic (P) material amongst A, B and C.



(a) [A - F, B - D, C - P]
(b) [A - F, B - P, C - D]
(c) [A - P, B - D, C - F]
(d) [A - D, B - F, C - P]

8. A, B and C are parallel conductors of equal lengths carrying currents I, I and $2I$ respectively. Distance between A and B is x. Distance between B and C is also x. F_1 is the force exerted by B on A. F_2 is the force exerted by conductor C on A. Choose the correct answer.
(a) $F_1 = 2F_2$ (b) $F_2 = 2F_1$
(c) $F_1 = F_2$ (d) $F_1 = -F_2$

9. A current-carrying loop is placed in a uniform magnetic field. The torque acting on it does not depend upon
(a) area of loop (b) no. of turns in loop
(c) shape of loop
(d) strength of current and magnetic field

10. A coil of N turns and radis R carries a current I. it is unwound and rewound to make another coil of radius $R/2$, current remaining the same. What is the ratio of the magnetic moments of the new coil and the original coil?
(a) 1 : 1 (b) 1 : 2 (c) 2 : 1 (d) 1 : 3

11. Two short bar magnets P and Q are placed one over another, with their magnetic axes mutually perpendicular to each other. Compare their magnetic moments if their resultant at a point (A) on the magnetic axis of P is inclined at 30° with the axis.
(a) $\frac{M_1}{M_2} = \frac{\sqrt{3}}{2}$ (b) $\frac{M_1}{M_2} = \frac{2}{\sqrt{3}}$
(c) $\frac{M_1}{M_2} = \frac{1}{\sqrt{3}}$ (d) $\frac{M_1}{M_2} = \frac{\sqrt{3}}{1}$

12. The magnetic susceptibility of a paramagnetic material at –73°C is 0.0075 and its value at –173°C will be
(a) 0.0030 (b) 0.0075
(c) 0.0045 (d) 0.015

13. A proton beam enters a magnetic field of 10^{-4} Wb m^{-2} normally. If the specific charge of the proton is 10^{11} C kg^{-1} and its velocity is 10^9 m s^{-1}, then the radius of the circle described will be
(a) 10 m (b) 1 m (c) 0.1 m (d) 100 m

14. A bar magnet 30 cm long is placed in the magnetic meridian with its north pole pointing geographical south. The neutral point is found at a distance of 30 cm from its centre. Find the pole strength of the magnet. (Take B_H = 0.34 G)
(a) 0.34 A m (b) 3.06 A m
(c) 4.93 A m (d) 8.61 A m

15. A charge q moves along axis of a current carrying solenoid with velocity $\vec{v}$. When charge enters inside the solenoid, its velocity become $\vec{v}'$. Then
(a) $\vec{v} = \vec{v}'$ and $|\vec{v}| \neq |\vec{v}'|$
(b) $\vec{v} \neq \vec{v}'$ and $|\vec{v}| \neq |\vec{v}'|$
(c) $\vec{v} \neq \vec{v}'$ and $|\vec{v}| = |\vec{v}'|$
(d) $\vec{v} = \vec{v}'$ and $|\vec{v}| = |\vec{v}'|$

16. Two long parallel wires carry currents i_1 and i_2 such that $i_1 > i_2$. When the currents are in the same

direction, the magnetic field at a point midway between the wires is 6×10^{-6} T. If the direction of i_2 is reversed, the field becomes 3×10^{-5} T. What is the ratio $\frac{i_1}{i_2}$?

(a) 1/2 (b) 2/3 (c) 3/4 (d) 3/2

17. A long straight wire of radius R carries a current I. The current is uniformly distributed across its cross section. What is the ratio of the magnetic fields at distance of $R/2$ and $2R$ from its axis?

(a) 1/2 (b) 1 (c) 2 (d) 4

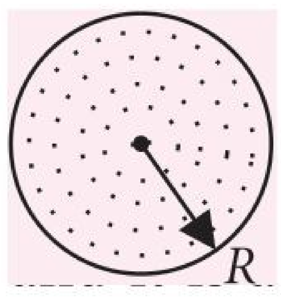
18. Two concentric coils each of radius equal to 2π cm are placed right angles to each other. If 3 A and 4 A are the currents flowing through the two coils respectively. The magnetic induction (in Wb m^{-2}) at the centre of the coils will be

(a) 10^{-5} (b) 7×10^{-5}
(c) 12×10^{-5} (d) 5×10^{-5}

19. A proton and an alpha particle both enter a region of uniform magnetic field B, moving at right angles to the field B. If the radius of circular orbits for both the particles is equal and the kinetic energy acquired by proton is 1 MeV, the energy acquired by the alpha particle will be

(a) 1.5 MeV (b) 1 MeV
(c) 4 MeV (d) 0.5 MeV

20. In the diagram shown, a time varying non uniform magnetic field passes through a circular region of radius R. The magnetic field is directed outwards function of radial distance r and time t according to the relation $B = B_0rt$. The induced electric field strength at a radial distance $R/2$ from the centre is



(a) $\frac{B_0R^2}{12}$ (b) $\frac{B_0R^2}{6}$ (c) $\frac{2B_0R^2}{3}$ (d) $\frac{B_0R^2}{8}$

SOLUTIONS

1. (b): The length of π^2 m is nearly 10 m and is large compared to 1 cm. Hence, the wire can be treated as infinitely long. Magnetic field at distance of 1 cm from the wire is

$$B_1 = \frac{\mu_0 I}{2\pi r} = \frac{\mu_0 I}{2\pi \times 1 \times 10^{-2}} = \frac{\mu_0 I}{2\pi} \times 10^2$$

When it is bent into a circle,
$2\pi r$ = circumference = length,

$$\therefore \quad 2\pi r = \pi^2 \Rightarrow r = \frac{\pi}{2}$$

Magnetic induction at the centre is

$$B_2 = \frac{\mu_0 nI}{2r} = \frac{\mu_0 I}{2\left(\frac{\pi}{2}\right)} = \frac{\mu_0 I}{\pi} \qquad \text{(since } n = 1\text{)}$$

$$\frac{B_1}{B_2} = \frac{\left(\frac{\mu_0 I}{2\pi}\right) \times 10^2}{\left(\frac{\mu_0 I}{\pi}\right)} = \frac{100}{2} = 50 \Rightarrow \frac{B_2}{B_1} = \frac{1}{50}$$

2. (d) **3. (c)**

4. (c) : The circulating electrons is equivalent to a circular current loop carrying a current I.

$$I = \frac{dQ}{dt} = \frac{e}{T} = \frac{e}{1/n} = ne$$

$= 6.25 \times 10^{15} \times 1.6 \times 10^{-19} = 10 \times 10^{-4} = 10^{-3}$ A

and B at the centre of the circular orbit $= \frac{\mu_0}{4\pi}\left(\frac{2\pi ni}{r}\right)$

In this case $n = 1$ and $r = 0.5 \times 10^{-10}$ m

$$\therefore \quad B = \frac{10^{-7} \times 2 \times 3.14 \times 10^{-3}}{\frac{1}{2} \times 10^{-10}}$$

$\therefore \quad B = 4 \times 3.14 = 12.56$ Wb m^{-2}

5. (c) : The charged particle will be accelerated along the y-axis, as the electric field acts along y-axis.

$$\therefore \quad a_y = \frac{\text{Force due to electric field}}{\text{Mass}} = \frac{qE}{m}$$

$$= \frac{10^{-6} \times 10^3}{10^{-3}} = 1 \text{ m s}^{-2} \qquad \left[\because \ E = \frac{F}{q}\right]$$

$\therefore$ Vertical velocity of the particle after 10 second is $v_y = a_y t = 1 \times 10 = 10$ m s^{-1}
and the horizontal velocity v_x remains constant and $v_x = 10$ m s^{-1}

$\therefore$ Resultant velocity $= \sqrt{v_x^2 + v_y^2} = \sqrt{10^2 + 10^2}$

$\therefore \quad v = 10\sqrt{2}$ m s^{-1}

6. (b): For the solenoid $B = \mu_0 nI$

$$= 4\pi \times 10^{-7} \times \frac{500}{0.4} \times 3 = 1.5\pi \times 10^{-3} \text{ T}$$

Dipole moment of the coil (M) = $NIA = NI\pi R^2$
$= 10 \times 0.4 \times \pi \times (10^{-2})^2$

$\therefore \quad M = 4\pi \times 10^{-4}$ Am^2

Torque required = $MB \sin 90° = MB$
$= 4\pi \times 10^{-4} \times 1.5\pi \times 10^{-3} = 6 \times 10^{-6}$ N m

7. (b): The slope of the graph of I versus H gives the susceptibility $\left(\chi = \frac{I}{H}\right)$.

∴ For C, χ is small but –ve, hence it is diamagnetic. For B, χ is small but +ve, it may be paramagnetic. The slope of A is +ve and its value is more than that of B.

∴ $\chi_B > \chi_A$ and it is +ve.

A will be ferromagnetic.

Thus correct option is [A - F, B - P, C - D].

8. (d): (i) Currents I and I flow through A and B, in the same direction

Let l be the length of A, B and C

∴ The force F_1 exerted by B on A is

$$F_1 = f_{B_A} = \frac{\mu_0 I^2 l}{2\pi x} \quad \text{(Force of attraction)}$$

(ii) Currents in A and C flow in opposite directions. Hence there is a force of repulsion.

$$F_2 = f_{C_A} = \frac{\mu_0 I(2I)l}{2\pi(2x)} = \frac{\mu_0 I^2 l}{2\pi x} \quad \text{(Repulsive)}$$

∴ $F_{BA} = -F_{CA}$ or $F_1 = -F_2$

9. (c)

10. (b): The original magnetic moment of the coil

$M_1 = NIA = NI(\pi R^2)$ ...(i)

When it is rewound to make another coil of radius $\frac{1}{2}$, and the number of turns N', then

Length of the coil, $L = N \times 2\pi R = N' \times 2\pi\left(\frac{R}{2}\right)$

∴ $N' = 2N$

∴ The new magnetic moment M_2 is

$$M_2 = N'IA = (2N)I \cdot \pi\left(\frac{R}{2}\right)^2 = NI\left(\frac{\pi R^2}{2}\right) \quad \text{...(ii)}$$

$$\therefore \frac{M_2}{M_1} = \frac{NI\left(\frac{\pi R^2}{2}\right)}{NI(\pi R^2)} = \frac{1}{2}$$

11. (a)

12. (d): According to Curie's law, the magnetic susceptibility (χ) of a paramagnetic substance is inversely proportional to absolute temperature T

i.e., $\chi \propto \frac{1}{T}$ ∴ $\frac{\chi_1}{\chi_2} = \frac{T_2}{T_1}$ or $\chi_2 = \chi_1 \frac{T_1}{T_2}$

Here,

$\chi_1 = 0.0075$

$T_1 = -73°C = (273 - 73)$ K = 200 K

$T_2 = -173°C = (273 - 173)$ K = 100 K

$$\therefore \chi_2 = (0.0075)\left(\frac{200\text{ K}}{100\text{ K}}\right) = 0.015$$

13. (d)

14. (d): Here the neutral point lie on the axial line.

$$\therefore B_{\text{axial}} = B_H \text{ or } \frac{\mu_0}{4\pi} \cdot \frac{2mr}{(r^2 - l^2)^2} = B_H$$

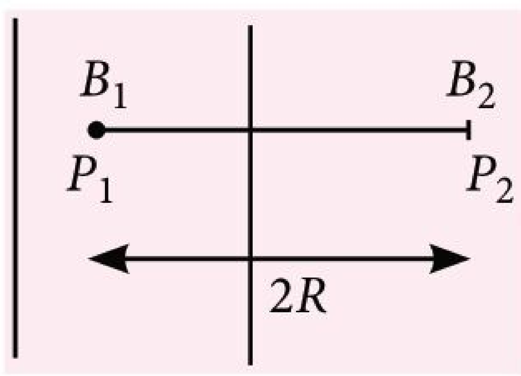
$$\text{or } m = \frac{4\pi}{\mu_0} \cdot \frac{B_H(r^2 - l^2)^2}{2r} = 2.582 \text{ A m}^2$$

Pole strength, $q_m = \frac{m}{2l} = \frac{2.582}{0.30} = 8.61$ A m

15. (d): Since $\vec{v}$ is parallel to $\vec{B}$, $\vec{F}_m = q(\vec{v} \times \vec{B}) = \vec{0}$ and hence $\vec{v}$ remains unchanged, *i.e.,* $\vec{v} = \vec{v}'$ and as such $|\vec{v}| = |\vec{v}'|$.

16. (d)

17. (b): Let B_1 and B_2 be the fields at P_1 and P_2 As the current is uniformly distributed, the current density $J = \frac{I}{\pi R^2}$ remains constant.



From Ampere's circuital law, $\oint B \cdot dl = \mu_0 I_{\text{enclosed}}$

For $r < R$, $B \cdot 2\pi r = \mu_0 \cdot J \cdot \pi r^2$

$$\therefore B = \mu_0 J \cdot \frac{r}{2} = \mu_0 \frac{I}{\pi R^2} \times \frac{r}{2}$$

But $r = \frac{R}{2}$

$$\therefore B_1 = \frac{\mu_0 I}{\pi R^2} \times \frac{R}{2 \times 2} = \frac{\mu_0 I}{4\pi R} \quad \text{...(i)}$$

But for $r = 2R$, full current I is enclosed

$$\therefore B_2 = \frac{\mu_0}{4\pi}\left(\frac{2I}{r}\right) = \frac{\mu_0}{4\pi}\left(\frac{2I}{2R}\right) = \frac{\mu_0 I}{4\pi R} \quad \text{...(ii)}$$

$$\therefore \frac{B_1}{B_2} = 1$$

18. (d) **19. (b)**

20. (a): Flux for loop of radius $R/2$ at time t

$$\phi = \int_0^{R/2} B 2\pi r dr = B_0 t 2\pi\left[\frac{r^3}{3}\right]_0^{R/2}$$

$$= B_0 2\pi \frac{1}{3}\left(\frac{R^3}{8}\right)(t), \quad \text{As } \vec{E} \cdot d\vec{l} = \frac{-d\phi}{dt}$$

$$\therefore E(2\pi R/2) = -\frac{d\phi}{dt} = -B_0 2\pi \frac{R^3}{24}$$

$$\Rightarrow E = -\frac{B_0 R^2}{12}$$

CBSE warm-up!

CLASS-XII

TERM-I OBJECTIVE TYPE QUESTIONS*

Series 3 — Magnetic Effects of Current and Magnetism

GENERAL INSTRUCTIONS

1. The Question Paper contains three sections.
2. Section A has 25 questions. Attempt any 20 questions.
3. Section B has 24 questions. Attempt any 20 questions.
4. Section C has 6 questions. Attempt any 5 questions.
5. All questions carry equal marks.
6. There is no negative marking.

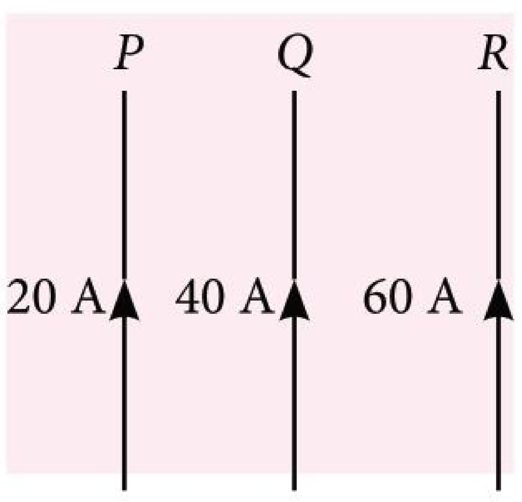
Time allowed : 90 minutes — ***Maximum marks : 35***

SECTION-A

This section consists of 25 multiple choice questions with overall choice to attempt any 20 questions. In case more than desirable number of questions are attempted, ONLY first 20 will be considered for evaluation.

1. A circular coil of 25 turns and radius 12 cm is placed in a uniform magnetic field of 0.5 T normal to the plane of the coil. If the current in the coil is 6 A then total torque acting on the coil is
(a) zero (b) 3.4 N m
(c) 3.8 N m (d) 4.4 N m

2. P, Q and R are long straight wires in air, carrying currents as shown in the figure. The force on Q is directed



(a) to the left
(b) to the right
(c) perpendicular to the plane of the diagram
(d) along the current in Q.

3. The force between two parallel current carrying wires is independent of
(a) their distance of separation
(b) the length of the wires
(c) the magnitude of currents
(d) the radii of the wires

4. Same current I passing through two parallel wires separated by a distance b, then force per unit length will be
(a) $\frac{\mu_0}{4\pi}\frac{2I^2}{b}$ (b) $\frac{\mu_0 I}{4\pi b^2}$ (c) $\frac{\mu_0 I^2}{4\pi b^2}$ (d) $\frac{\mu_0 I^2}{4\pi b}$

5. The magnetic dipole moment of a current carrying coil does not depend upon
(a) number of turns of the coil
(b) cross-sectional area of the coil
(c) current flowing in the coil
(d) material of the turns of the coil.

6. An electron is released from rest in a region of uniform electric and magnetic fields acting parallel to each other. The electron will
(a) move in a straight line.
(b) move in a circle.
(c) remain stationary.
(d) move in a helical path.

7. The magnet of pole strength m and magnetic moment M is cut into two pieces along its axis. Its pole strength and magnetic moment now become
(a) $\frac{m}{2}, \frac{M}{2}$ (b) $m, \frac{M}{2}$
(c) $\frac{m}{2}, M$ (d) m, M

**Chapterwise practice questions for CBSE Exam Term- I as per the pattern issued by CBSE.*

8. A circular coil of 20 turns and radius 10 cm is placed in a uniform magnetic field of 0.10 T normal to the plane of the coil. If the current in the coil is 5.0 A, the total torque on the coil will be

(a) 0 (b) 0.314 J
(c) 3.14 J (d) 6.28 J

9. A rectangular coil of length 0.12 m and width 0.1 m having 50 turns of wire is suspended vertically in a uniform magnetic field of strength 0.2 Wb m^{-2}. The coil carries a current of 2 A. If the plane of the coil is inclined at an angle of 30° with the direction of the field, the torque required to keep the coil in stable equilibrium will be

(a) 0.24 N m (b) 0.12 N m
(c) 0.15 N m (d) 0.20 N m

10. In an inertial frame of reference, the magnetic force on a moving charged particle is $\vec{F}$. Its value in another inertial frame of reference will be

(a) remain same.
(b) changed due to change in the amount of charge.
(c) changed due to change in velocity of charged particle.
(d) changed due to change in field direction.

11. The magnetic force $\vec{F}$ on a current carrying conductor of length l in an external magnetic field $\vec{B}$ is given by

(a) $\frac{I \times \vec{B}}{\vec{l}}$ (b) $\frac{\vec{l} \times \vec{B}}{I}$

(c) $I(\vec{l} \times \vec{B})$ (d) $I^2\vec{l} \times \vec{B}$

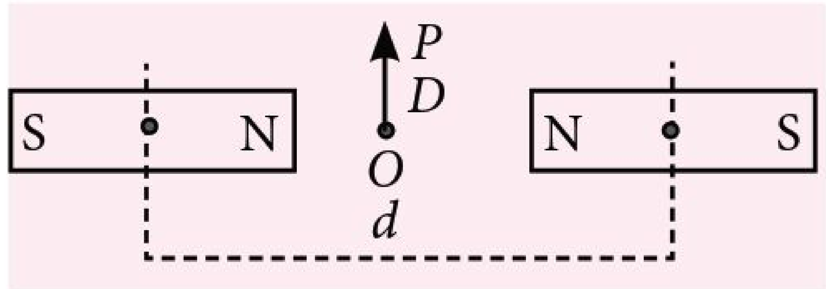
12. A bar magnet of pole strength 10 A m is cut into two equal parts breadthwise. The pole strength of each magnet is

(a) 5 A m (b) 10 A m (c) 15 A m (d) 20 A m

13. A bar magnet has a magnetic moment of 200 A m^2. This magnet is suspended in a magnetic field of 0.30 N A^{-1} m^{-1}. The torque required to rotate the magnet from its equilibrium position through an angle of 30°, will be

(a) 30 N m (b) $30\sqrt{3}$ N m
(c) 60 N m d) $60\sqrt{3}$ N m

14. Two identical bar magnets are fixed with their centres at a distance d apart. A stationary charge Q is placed at P in between the gap of the two magnets at a distance D from the centre O as shown in the figure



The force on the charge Q is

(a) zero (b) directed along OP
(c) directed along PO
(d) directed perpendicular to the plane of paper

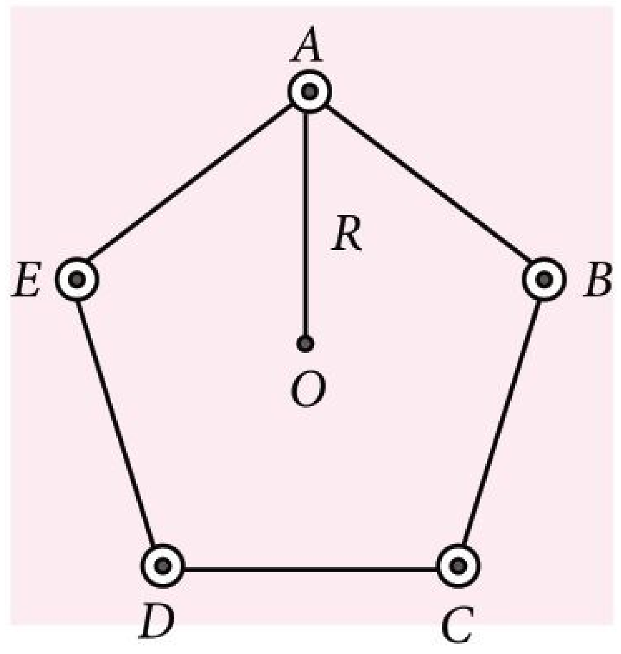
15. A charged particle is moving on a circular path with velocity v in a uniform magnetic field B. If the velocity of the charged particle is doubled and strength of magnetic field is halved, then radius becomes

(a) 8 times (b) 4 times
(c) 2 times (d) 16 times

16. The particle that cannot be accelerated by a cyclotron is

(a) proton (b) α-particle
(c) electron (d) deuteron nucleus.

17. Five long wires A, B, C, D and E each carrying current I are arranged to form edges of a pentagonal prism as shown in figure. Each wire carries current out of the plane of paper, the magnetic induction at a point on the axis O is



(axis O is at a distance R from each wire)

(a) equal to zero (b) less than zero
(c) more than zero (d) infinite

18. Two charged particles traverse identical helical paths in a completely opposite sense in a uniform magnetic field $\vec{B} = B_0\hat{k}$.

(a) They have equal z-components of momenta.
(b) They must have equal charges.
(c) They necessarily represent a particle-antiparticle pair.
(d) The charge to mass ratio satisfy.

$$\left(\frac{e}{m}\right)_1 + \left(\frac{e}{m}\right)_2 = 0.$$

19. At a given place on the earth's surface, the horizontal component of earth's magnetic field is 3×10^{-5} T and resultant magnetic field is 6×10^{-5} T. The angle of dip at this place is

(a) 30° (b) 40° (c) 50° (d) 60°

20. At two different places the angles of dip are respectively 30° and 45°. At these two places the

ratio of horizontal component of earth's magnetic field is

(a) $\sqrt{3}:\sqrt{2}$ (b) $1:\sqrt{2}$ (c) $1:2$ (d) $1:\sqrt{3}$

21. If the horizontal component of the earth's magnetic field is 0.30 G, and the dip angle is 60° at a given place, then the value of earth's total magnetic field is

(a) 0.15 G (b) $0.15\sqrt{3}$ G
(c) $0.15\sqrt{2}$ G (d) 0.60 G

22. Biot-Savart law indicates that the moving electrons (velocity $\vec{v}$) produce a magnetic field $\vec{B}$ such that

(a) $\vec{B} \perp \vec{v}$
(b) $\vec{B} \,||\, \vec{v}$
(c) it obeys inverse cube law.
(d) it is along the line joining the electron and point of observation.

23. A galvanometer of resistance 70 Ω, is converted to an ammeter by a shunt resistance r_s = 0.03 Ω. The value of its resistance will become

(a) 0.025 Ω (b) 0.022 Ω
(c) 0.035 Ω (d) 0.030 Ω

24. The cyclotron frequency υ_c is given by

(a) $\frac{qB}{2\pi m}$ (b) $\frac{mB}{2\pi q}$ (c) $\frac{2\pi m}{qB}$ (d) $\frac{2\pi B}{qm}$

25. In a cyclotron, a charged particle

(a) undergoes acceleration all the time.
(b) speeds up between the dees because of the magnetic field.
(c) speeds up in a dee.
(d) slows down within a dee and speeds up between dees.

SECTION-B

This section consists of 24 multiple choice questions with overall choice to attempt any 20 questions. In case more than desirable number of questions are attempted, ONLY first 20 will be considered for evaluation.

26. In a permanent magnet, at room temperature

(a) magnetic moment of each molecule is zero.
(b) the individual molecules have non-zero magnetic moment which are all perfectly aligned.
(c) domains are partially aligned.
(d) domains are all perfectly aligned.

27. At certain place, the horizontal component of earth's magnetic field is 3.0 G and the angle dip at that place is 30°. The magnetic field of earth at that location

(a) 4.5 G (b) 5.1 G (c) 3.5 G (d) 6.0 G

28. The angles of dip at the poles and the equator respectively are

(a) 30°, 60° (b) 0°, 90°
(c) 45°, 90° (d) 90°, 0°

29. A jet plane is travelling west at 450 metre per second. If the horizontal component of earth's magnetic field is 4×10^{-4} Tesla and angle of dip is 30°, then its vertical component is

(a) 3×10^{-4} T (b) 4×10^{-4} T
(c) 2.3×10^{-4} T (d) 12.308×10^{-4} T

30. The angle between the true geographic north and the north shown by a compass needle is called as

(a) Inclination (b) Magnetic declination
(c) Angle of meridian (d) Magnetic pole

31. To produce a magnetic field of π tesla at the centre of circular loop of diameter 1 m, the current flowing through loop is

(a) 5×10^6 A (b) 10^7 A
(c) 2.5×10^6 A (d) 2×10^6 A

32. A circular current carrying coil has a radius R. The distance from the centre of the coil on the axis where the magnetic induction will be $(1/8)^{th}$ of its value at the centre of the coil, is

(a) $R/\sqrt{3}$ (b) $R\sqrt{3}$
(c) $2R\sqrt{3}$ (d) $(2/\sqrt{3})R$

33. A charge q moves with a velocity 2 m s^{-1} along x-axis in a uniform magnetic field $\vec{B} = (\hat{i} + 2\hat{j} + 3\hat{k})$ T, then charge will experience a force

(a) in zy plane (b) along $-y$ axis
(c) along $+z$ axis (d) along $-z$ axis

34. At a certain location in Africa, compass points 12° west of geographic north. The north tip of magnetic needle of a dip circle placed in the plane of magnetic meridian points 60° above the horizontal. The horizontal component of earth's field is measured to be 0.16 G. The magnitude of earth's field at the location is

(a) 0.32 G (b) 0.42 G (c) 4.2 G (d) 3.2 G

35. In the magnetic meridian of a certain place the horizontal component of earth's magnetic field is 0.25 G and dip angle is 60°. The magnetic field of the earth at this location is

(a) 0.50 G (b) 0.52 G (c) 0.54 G (d) 0.56 G

36. The equatorial magnetic field of earth is 0.4 G. Then its dipole moment on equator is

(a) 1.05×10^{23} A m^2 (b) 2.05×10^{23} A m^2
(c) 1.05×10^{21} A m^2 (d) 2.05×10^{21} A m^2

37. Which one of the following is not correct about Lorentz force?

(a) In presence of electric field $\vec{E}(r)$ and magnetic field $\vec{B}(r)$ the force on a moving electric charge is $\vec{F} = q[\vec{E}(r) + \vec{v} \times \vec{B}(r)]$.
(b) The force, due to magnetic field on a negative charge is opposite to that on a positive charge.
(c) The force due to magnetic field become zero if velocity and magnetic field are parallel or anti-parallel.
(d) For a static charge the magnetic force is maximum.

38. A straight wire having mass of 1.2 kg and length of 1 m carries a current of 5 A. If the wire is suspended in mid-air by a uniform horizontal magnetic field, then the magnitude of field is

(a) 0.65 T (b) 1.53 T (c) 2.4 T (d) 3.2 T

39. A 2.5 m long straight wire having mass of 500 g is suspended in mid air by a uniform horizontal magnetic field *B*. If a current of 4 A is passing through the wire then the magnitude of the field is (Take $g = 10$ m s^{-2})

(a) 0.5 T (b) 0.6 T (c) 0.25 T (d) 0.8 T

40. A current of 10 A is flowing in a wire of length 1.5 m. A force of 15 N acts on it when it is placed in a uniform magnetic field of 2 T. The angle between the magnetic field and the direction of the current is

(a) 30° (b) 45° (c) 60° (d) 90°

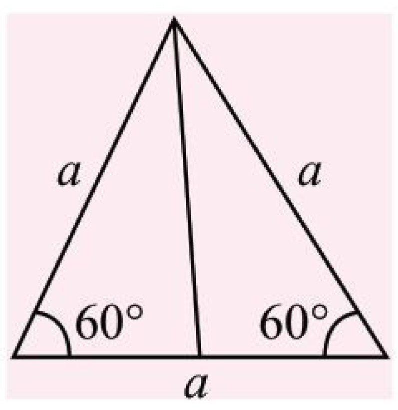
41. The magnetic force per unit length on a wire carrying a current of 10 A and making an angle of 45° with the direction of a uniform magnetic field of 0.20 T is

(a) $2\sqrt{2}$ N m^{-1} (b) $\frac{2}{\sqrt{2}}$ N m^{-1}
(c) $\frac{\sqrt{2}}{2}$ N m^{-1} (d) $4\sqrt{2}$ N m^{-1}

42. Magnetic moment for a solenoid and corresponding bar magnet is

(a) equal for both (b) more for solenoid
(c) more for bar magnet (d) none of these

43. A uniform conducting wire of length length $12a$ and resistance *R* is wound up into four turn as a current carrying coil in the shape of equilateral triangle of side *a*. If current *I* is flowing through the coil then the magnetic moment of the coil is



(a) $\frac{\sqrt{3}}{2}a^2 I$ (b) $\frac{a^2 I}{\sqrt{3}}$ (c) $\sqrt{3}a^2 I$ (d) $\frac{2a^2 I}{\sqrt{3}}$

44. Magnetic field at the centre of a circular loop of area *A* is *B*. The magnetic moment of the loop is

(a) $\frac{BA^2}{\mu_0 \pi}$ (b) $\frac{BA\sqrt{A}}{\mu_0}$
(c) $\frac{BA\sqrt{A}}{\mu_0 \pi}$ (d) $\frac{2BA\sqrt{A}}{\mu_0 \sqrt{\pi}}$

45. Given below are two statements labelled as Assertion (A) and Reason (R)

Assertion (A) : The magnetic field intensity at the centre of a circular coil carrying current changes, if the current through the coil is doubled.

Reason (R) : The magnetic field intensity is dependent on current in conductor.

Select the most appropriate answer from the options given below:

(a) Both A and R are true and R is the correct explanation of A.
(b) Both A and R are true but R is not the correct explanation of A.
(c) A is true but R is false.
(d) A is false and R is also false.

46. Given below are two statements labelled as Assertion (A) and Reason (R)

Assertion (A) : A linear solenoid carrying current is equivalent to a bar magnet.

Reason (R) : The magnetic field lines of both are same.

Select the most appropriate answer from the options given below:

(a) Both A and R are true and R is the correct explanation of A.
(b) Both A and R are true but R is not the correct explanation of A.
(c) A is true but R is false.
(d) A is false and R is also false.

47. Given below are two statements labelled as Assertion (A) and Reason (R)

Assertion (A) : Vibration magnetometer can be used to compare the magnetic moment of two bar magnets.

Reason (R) : By using vibration magnetometer we can find the time period of vibration of the magnet.

Select the most appropriate answer from the options given below:

(a) Both A and R are true and R is the correct explanation of A.
(b) Both A and R are true but R is not the correct explanation of A.
(c) A is true but R is false.
(d) A is false and R is also false.

48. **Given below are two statements labelled as Assertion (A) and Reason (R)**

Assertion (A) : Free electrons always keep on moving in a conductor even then no magnetic force act on them in magnetic field unless a current is passed through it.

Reason (R) : The average velocity of free electron is zero.

Select the most appropriate answer from the options given below:

(a) Both A and R are true and R is the correct explanation of A.
(b) Both A and R are true but R is not the correct explanation of A.
(c) A is true but R is false.
(d) A is false and R is also false.

49. **Given below are two statements labelled as Assertion (A) and Reason (R)**

Assertion (A) : The ion cannot move with a speed beyond a certain limit in a cyclotron.

Reason (R) : As velocity increases time taken by ion increases.

Select the most appropriate answer from the options given below:

(a) Both A and R are true and R is the correct explanation of A.
(b) Both A and R are true but R is not the correct explanation of A.
(c) A is true but R is false.
(d) A is false and R is also false.

SECTION-C

This section consists of 6 multiple choice questions with an overall choice to attempt any 5. In case more than desirable number of questions are attempted, ONLY first 5 will be considered for evaluation.

50. An arc of a circle of radius R subtends an angle $\frac{\pi}{2}$ at the centre. It carries a current I. The magnetic field at the centre will be

(a) $\frac{\mu_0 I}{2R}$ (b) $\frac{\mu_0 I}{8R}$
(c) $\frac{\mu_0 I}{4R}$ (d) $\frac{2\mu_0 I}{5R}$

51. A steel wire of length l has a magnetic moment M. It is then bent into a semicircular arc. The new magnetic moment is

(a) M (b) $\frac{2M}{\pi}$ (c) $\frac{M}{l}$ (d) $M \times l$

Case Study : Read the following paragraph and answer the questions :

Force on a charged particle in a uniform electric and magnetic field

When a charged particle of charge q moving into a uniform electric field $\vec{E}$, the force acting on it is given by $\vec{F} = q\vec{E}$.

The direction of $\vec{F}$ is same as that of $\vec{E}$ is positive if q is +ve and $-\vec{E}$ if q is –ve.

When a charged particle of charge q, moving with velocity $\vec{v}$ is subjected to a uniform magnetic field $\vec{B}$, the force acting on it is $\vec{F} = q(\vec{v} \times \vec{B})$; $F = qvB \sin\theta$ where θ is the angle between $\vec{v}$ and $\vec{B}$.

The direction of this force is perpendicular to the plane containing $\vec{v}$ and $\vec{B}$.

$\vec{F} = 0$ if $\vec{v} = 0$, *i.e.*, a charge at rest does not experience any magnetic force.

$\vec{F} = 0$ if $\theta = 0$ or $180°$ *i.e.*, the magnetic force vanishes if $\vec{v}$ is either parallel or antiparallel to the direction of $\vec{B}$

Force will be maximum if θ = 90°, *i.e.*, if $\vec{v}$ is perpendicular to $\vec{B}$, the magnetic force has a maximum value and is given by $F_{max} = qvB$.

The following questions are multiple choice questions. Choose the most appropriate answer :

52. A proton, a deuteron and an α-particle accelerated through the same potential difference enter a region of uniform magnetic field, moving at right angles to B. What is the ratio of their K.E.?

(a) 2 : 1 : 1 (b) 2 : 2 : 1
(c) 1 : 2 : 1 (d) 1 : 1 : 2

53. An α particle and a proton having same momentum enter into a region of uniform magnetic field and move in circular paths. The ratio of the radii of curvature of their paths, R_α/R_b in the field is

(a) 1/2 (b) 1/4 (c) 1 (d) 4

54. Electrons moving with different speeds enter a uniform magnetic field in a direction perpendicular to the field. They will move along circular paths

(a) of the same radius
(b) with larger radii for the faster electrons
(c) with smaller radii for the faster electrons
(d) either (b) or (c) depending on the magnitude of the magnetic field.

55. A strong magnetic field is applied on a stationary electron. Then the electron

(a) moves in the direction of the field
(b) remains stationary
(c) moves perpendicular to the direction of the field
(d) moves opposite to the direction of the field

SOLUTIONS

1. (a) : The torque acting on the coil

$|\vec{\tau}| = |\vec{m} \times \vec{B}| = mB\sin\theta$

Here the circular coil is placed normal to the direction of magnetic field then the angle between the direction of magnetic moment $(\vec{m})$ and magnetic field $(\vec{B})$ is zero, then $\tau = mB\sin\theta = mB\sin 0 = 0 \quad \therefore \quad \tau = 0$

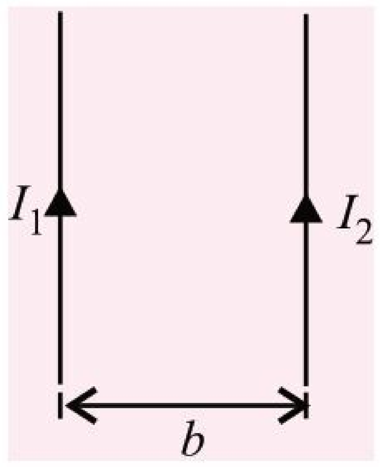
2. (a) : Parallel currents attract and antiparallel currents repel.

Due to both 20 A (P) as well as 60 A (R), the force on Q is towards left.

3. (d) The force between two parallel current carrying wires is independent of the radii of the wires.

4. (a) : Force per unit length between two parallel current carrying wires separated by a distance b

$= \frac{\mu_0}{4\pi}\frac{2I_1I_2}{b} = \frac{\mu_0}{4\pi}\frac{2I^2}{b} \quad (\because I_1 = I_2 = I)$



5. (d) : Magnetic dipole moment of a curret carrying coil, $M = NIA$

It does not depend upon the material of the turns of the coil

6. (a)

7. (a) : When cut along the axis, area of cross-section becomes half. Therefore, pole strength is halved and $M = m\,(2l)$, is also halved.

8. (a) : Total torque on the coil, $\tau = BIAN\sin\theta$

where θ is the angle which the normal to the plane of the coil makes with the direction of magnetic field.

Here, $\theta = 0° \quad \therefore \quad \tau = 0$

9. (d) : The required torque is, $\tau = NIAB\sin\theta$

Here, $N = 50$, $I = 2$ A, $A = 0.12\text{ m} \times 0.1\text{ m} = 0.012\text{ m}^2$

$B = 0.2\text{ Wb m}^{-2}$ and $\theta = 90° - 30° = 60°$

$\therefore \quad \tau = (50)(2\text{ A})(0.012\text{ m}^2)(0.2\text{ Wb m}^{-2})\sin 60°$

$= 0.20$ N m

10. (c) : $\vec{F} = q(\vec{v} \times \vec{B}) \quad \therefore \quad F = qvB\sin\theta$

which shows magnetic field is velocity dependent due to which it differs from one inertial frame to another.

11. (c)

12. (b) : The pole strength of bar magnet = 10 A m

Since, if bar magnet is cut into equal parts breadthwise then the pole strength of each magnets will always be same. *i.e.* 10 A m.

13. (a) : Torque experienced by a magnet suspended in a uniform magnetic field B is given by $\tau = MB\sin\theta$

Here, $M = 200\text{ A m}^2$, $B = 0.30\text{ N A}^{-1}\text{ m}^{-1}$ and $\theta = 30°$

$\therefore \quad \tau = 200 \times 0.30 \times \sin 30°$

$\tau = 30$ N m

14. (a) : Magnetic field due to bar magnets exerts force on moving charges only. Since the charge is at rest, zero force acts on it.

15. (b) : $Bqv = \frac{mv^2}{r} \Rightarrow r = \frac{mv}{Bq}$

According to statement, $v' = 2v$ and $B' = \frac{B}{2}$

$\therefore \quad r' = \frac{mv'}{B'q} = \frac{m(2v)}{(B/2)q} = \frac{4mv}{Bq} = 4r$

16. (c) : Cyclotron is suitable only for accelerating heavy particles like proton, deuteron, α-particle etc. Electrons cannot be accelerated by the cyclotron because the mass of the electron is small and a small increase in energy of the electron makes the electrons move with a very high speed. As a result of it, the electrons go quickly out of step with oscillating electric field.

17. (a) : Magnetic field induction due to five wires will be represented by various sides of closed pentagon in one order, lying in the plane of paper. So, its value is zero.

18. (d)

19. (d) : As $B_H = B\cos\delta$

$\therefore \quad \cos\delta = \frac{B_H}{B} = \frac{3\times10^{-5}}{6\times10^{-5}} = \frac{1}{2}$; $\delta = 60°$

20. (a) : Horizontal component, $B_H = B\cos\theta$

$\therefore \quad B_{H1} = B\cos 30°$ and $B_{H2} = B\cos 45°$

Thus,

$$\frac{B_{H_1}}{B_{H_2}} = \frac{\cos 30°}{\cos 45°} = \frac{\frac{\sqrt{3}}{2}}{\frac{1}{\sqrt{2}}} = \frac{\sqrt{3}}{\sqrt{2}}$$

21. (d) : $B_H = 0.30$ G

Angle of dip, $\delta = 60°$

Let earth's total magnetic field = B

Then, $B\cos\delta = B_H$

$\Rightarrow \quad B = \frac{B_H}{\cos\delta} = \frac{0.30}{\cos 60°} = 0.60$ G

22. (a) : $\vec{B} \perp \vec{v}$

Magnetic field produced by charges moving with velocity $\vec{v}$, at a distance r is

$$\vec{B} = \left(\frac{\mu_0}{4\pi}\right)\cdot q\left(\frac{\vec{v}\times\vec{r}}{r^2}\right)$$

Therefore, $\vec{B} \perp \vec{v}$

23. (d) : Here, $R_g = 70\ \Omega$; $r_s = 0.03\ \Omega$

$R = \frac{R_g r_s}{R_g + r_s} \quad \therefore \quad R = \frac{70 \times 0.03}{70 + 0.03} \approx 0.03\ \Omega$

24. (a) : The cyclotron frequency υ_c is given by $\frac{qB}{2\pi m}$.

25. (a) : In a cyclotron, charged particle experiences coulombic force between the Dees and magnetic force while circulating inside the Dees, *i.e.*, it always experiences a force.

26. (c) : A permanent magnet at room temperature retains a ferromagnetic properly that possesses a dipole moment. And domains are partially aligned due to thermal agitation.

27. (c) : Given $B_H = B\cos\theta = 3$ G, $\theta = 30°$

$$B = \frac{B_H}{\cos 30°} = \frac{3}{\sqrt{3}/2} = 2\sqrt{3} = 3.5 \text{ G}$$

28. (d) : Since angle of dip at a place is defined as the angle δ, which is the direction of total intensity of earth's magnetic field B makes with a horizontal line in magnetic meridian,

At poles $B = B_V$ and $B_V = B\sin\delta$

$\therefore \sin\delta = 1 \Rightarrow \delta = 90°$

At equator $B = B_H$ and $B_H = B\cos\delta$

$\therefore \cos\delta = 1 \Rightarrow \delta = 0°$.

29. (c) : Given, $B_H = 4 \times 10^{-4}$ T, $\delta = 30°$

As $B_V = B_H \tan\delta = 4 \times 10^{-4} \times \tan 30° = 2.3 \times 10^{-4}$ T

30. (b) : The angle between the true geographic north and the north shown by a compass needle is called as magnetic declination or simply declination.

31. (c) : $B = \frac{\mu_0}{4\pi}\frac{2\pi I}{r}$

or $I = \frac{2Br}{\mu_0} = \frac{2\times\pi\times 0.5}{4\pi\times 10^{-7}} = 2.5\times 10^6$ A

32. (b) : $B_{\text{axis}} = \frac{\mu_0}{4\pi}\times\frac{2\pi IR^2}{(R^2+x^2)^{3/2}}$

At centre, $B_{\text{centre}} = \frac{\mu_0}{4\pi}\times\frac{2\pi I}{R}$

In the given problem,

$$\frac{\mu_0}{4\pi}\times\frac{2\pi IR^2}{(R^2+x^2)^{3/2}} = \frac{1}{8}\left[\frac{\mu_0}{4\pi}\times\frac{2\pi I}{R}\right]$$

or $(R^2 + x^2)^{3/2} = 8R^3$

Solving, we get $x = \sqrt{3}R$

33. (a) : $\vec{F} = q(\vec{v}\times\vec{B})$

$= q[(2\hat{i}\times(\hat{i}+2\hat{j}+3\hat{k})] = (4q)\hat{k} - (6q)\hat{j}$

So, the charge will experience force in *zy* plane.

34. (a) : Here, $B_H = 0.16$ G $= 0.16 \times 10^{-4}$ T, dip angle (δ) = 60°

Then, magnitude of earth's field.

$$B = \frac{B_H}{\cos\delta} = \frac{0.16\times 10^{-4}}{\cos 60°}\text{ T}$$

$$\Rightarrow B = \frac{0.16\times 10^{-4}}{1/2}$$

$= 0.32 \times 10^{-4}$ T = 0.32 G

35. (a) : Here, $B_H = 0.25$ G and $\cos\delta = \frac{B_H}{B}$

$\therefore$ The magnetic field of earth at the given location is

$$B = \frac{B_H}{\cos 60°} = \frac{0.25}{1/2} = 0.50 \text{ G}$$

36. (a) : Here, $B_H = 0.4$ G $= 0.4 \times 10^{-4}$ T, $r = 6.4 \times 10^6$ m

As $B_E = \frac{\mu_0}{4\pi}\frac{m}{r^3}$

$$\therefore \quad m = \frac{B_E r^3}{\mu_0/4\pi} = \frac{0.4\times 10^{-4}\times(6.4\times 10^6)^3}{10^{-7}}$$

$= 1.05 \times 10^{23}$ A m^2.

37. (d) : If charge is not moving then the magnetic force is zero.

Since $\vec{F}_m = q(\vec{v}\times\vec{B})$

As $\vec{v} = 0$, for stationary charge $\therefore \vec{F}_m = 0$

38. (c) : For mid-air suspension the upward force F on wire due to magnetic field B must be balanced by the force due to gravity, then

$IlB = mg$; $B = \frac{mg}{Il}$

Here, $m = 1.2$ kg, $g = 10$ m s^{-2}, $I = 5$ A, $l = 1$ m

$$B = \frac{1.2\times 10}{5\times 1} = 2.4 \text{ T}$$

39. (a) : Here, $m = 500$ g $= 0.5$ kg, $I = 4$ A, $l = 2.5$ m

As $F = IlB\sin\theta$

$mg = IlB \sin 90°$, $\quad(\because \theta = 90° \text{ and } F = mg)$

$$\therefore \quad B = \frac{mg}{Il} = \frac{0.5\times 10}{4\times 2.5} = 0.5 \text{ T}$$

40. (a) : $F = IlB\sin\theta$ or $\sin\theta = \frac{F}{IlB}$

$$\sin\theta = \frac{15}{10\times 1.5\times 2} = \frac{1}{2} \quad\text{or}\quad \theta = 30°$$

41. (b) : $I = 10$ A, $\theta = 45°$, $B = 0.2$ T

$\therefore F = IlB\sin\theta$

$$\therefore \quad \frac{F}{l} = IB\sin 45° = 10\times 0.2\times\frac{1}{\sqrt{2}} = \frac{2}{\sqrt{2}} \text{ N m}^{-1}$$

42. (a) : Since a bar magnet and a corresponding solenoid produce similar magnetic fields. Hence the magnetic moment of a bar magnet is equal to the magnetic moment of an equivalent solenoid that produces the same magnetic field.

43. (c) : Magnetic moment $M = NIA$

$$= 4\times I\times a^2\frac{\sqrt{3}}{4} = \sqrt{3}a^2 I$$

44. (d) : Let r be the radius of the circular loop.

$$\therefore \quad A = \pi r^2 \text{ or } r = \sqrt{\frac{A}{\pi}}$$

Magnetic field at the centre of the loop is

$$B = \frac{\mu_0 I}{2r} = \frac{\mu_0 I}{2\sqrt{\frac{A}{\pi}}} \quad \text{or} \quad I = \frac{2B}{\mu_0}\sqrt{\frac{A}{\pi}}$$

The magnetic moment of the loop is

$$M = IA = \frac{2B}{\mu_0}\sqrt{\frac{A}{\pi}}A = \frac{2BA\sqrt{A}}{\mu_0\sqrt{\pi}}$$

45. (a) : The magnetic field at the centre of circular coil is given by $B = \frac{\mu_0}{4\pi}\frac{2\pi nI}{a}$.

So if current through coil is doubled then magnetic field is $B' = 2B$.

The magnetic field also get doubled. The magnetic field is directly proportional to the current in conductor.

46. (a) : The magnetic lines of force due to current carrying straight solenoid is same as that of bar magnet (That's why at some places solenoid is used as temporary magnet).

47. (b) : Using vibration magnetometer we can find time period of vibration of magnet which is given by

$$\Rightarrow T = 2\pi\sqrt{\frac{I}{MB_H}}$$

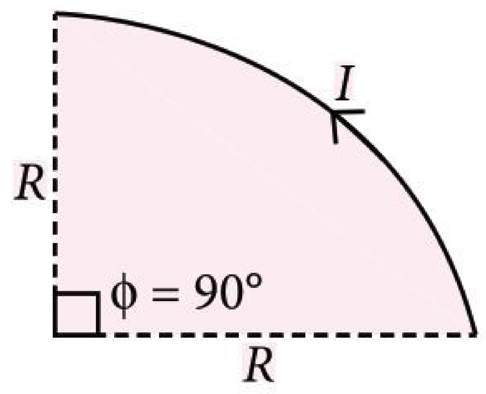
Where M = magnetic dipole moment

BH = horizontal component of earth's magnetic field. Using magnetometer we can find out time period of two magnets. Using above formula we can compare their magnetic moment by $\frac{M_1}{M_2} = \frac{T_2^2}{T_1^2}$.

48. (c) : In the absence of the electric current, the free electrons in a conductor are in a state of random motion, like molecule in a gas. Their average velocity is zero, *i.e.*, they do not have any net velocity in a direction. As a result, there is no net magnetic force on the free electrons in the magnetic field. On passing the current, the free electrons acquire drift velocity in a definite direction, hence magnetic force acts on them, unless the field has no perpendicular component.

49. (a) : As velocity increases, the positive ion will take longer time to describe semicircular path than the time for half cycle of oscillating electric field. As a result of it the ion will not arrive in the gap between the two dees exactly at the instant, the polarity of the two dees in reversed and hence will not be accelerated further.

50. (b) : Magnetic field at the centre of a circular arc of radius R, carries current I and making an angle ϕ at the centre is given by



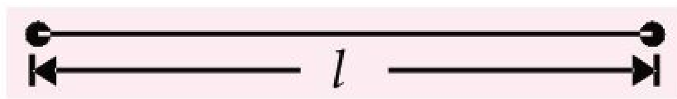
$$B = \frac{\mu_0 I\phi}{4\pi R}$$

In the given problem $\phi = \frac{\pi}{2}$

$$\therefore \quad B = \frac{\mu_0 I(\pi/2)}{4\pi R} = \frac{\mu_0 I}{8R}$$

51. (b) : Magnetic moment of a wire of length l is

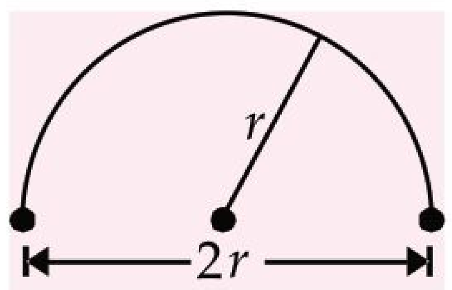
$M = ml$ ...(i)



where m is the strength of each pole.

When the wire is bent into a semicircle of radius r, then

$$l = \pi r \text{ or } r = \frac{l}{\pi}$$



Now, the distance between two poles

$$= 2r = \frac{2l}{\pi}$$

Its new magnetic moment is

$$M' = m\times 2r = m\times\frac{2l}{\pi} = \frac{2}{\pi}(m\times l) = \frac{2M}{\pi} \qquad \text{(Using(i))}$$

52. (d) : K.E. gained by charged particle of charge q when accelerated under a potential difference V will be $K = qV$.

For a given V, $K \propto q$.

For proton, deuteron and α-particle, the ratio of charges is 1 : 1 : 2.

53. (a) : α-particle = $_2He^4$

$\therefore$ charge = $2e$,

Proton = $_1H^1$, $\therefore$ charge = e

Momentum, mava = $m_p v_p$

For circular motion in uniform magnetic field,

$$Bqv = \frac{mv^2}{R} \text{ or } BqR = mv \quad \therefore \quad Bq\text{a}R\text{a} = Bq_pR_p$$

$$\text{or } \frac{R_\alpha}{R_p} = \frac{q_p}{q_\alpha} = \frac{e}{2e} = \frac{1}{2} \quad \therefore \quad \frac{R_\alpha}{R_p} = \frac{1}{2}$$

54. (b) : $\frac{mv^2}{r} = Bqv$ or $mv = Bqr$ or $r = \left(\frac{m}{Bq}\right)v$

As m, q and B are the same for all the electrons, $r \propto v$.

55. (b) : As $\vec{F} = \vec{q}(\vec{v}\times\vec{B})$

As the electron is stationary, $\therefore$ velocity $\vec{v} = 0$.

$\therefore$ $\vec{F} = 0$. So, electron will remain stationary.